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TEST DESCRIPTION AND SAMPLE QUESTIONS FOR UT AUSTIN TESTS FOR CREDIT IN PHYSICS

Part I covers mechanics, fluids and waves. Part II covers electricity, magnetism and optics. The total working time for each part is 90 minutes.

The use of a hand calculator or any formula sheet is not permitted for any part of the examination. The questions are designed and graded to minimize the amount of arithmetic computation required. A formula sheet is included with this test description to illustrate the level of difficulty and to provide some indication of the material covered. This formula sheet is not to be brought to the examination and will not be available during the examination.

Calculus is used freely in formulating principles and in solving problems.

Essential elements to be studied in preparation for Part I.

Mechanics Fluids and Waves

- Vectors (vector algebra, components, coordinate systems)
- Kinematics (calculus of velocity and acceleration vectors, projectile motion, circular motion)
- Particle dynamics (Newton's laws and their application, frictional forces, centripetal forces)
- Work (integral form of the work-energy theorem, potential energy, conservative forces)
- Momentum (conservation of momentum, center of mass)
- Collisions (impulse, isolated systems, and momentum conservation; elastic and inelastic collisions)
- Rigid body motion (torque, moment of inertia, angular momentum, and its conservation)
- Gravitation and planetary motion
- Oscillations (simple harmonic motion, energy in oscillations)
- Fluids (static and dynamic)
- Waves (mechanical and sound)

Essential elements to be studied in preparation for Part II.

Electricity, Magnetism and Optics

The primary emphasis is on classical electricity and magnetism.

- Electric charge, Coulomb's law, and electric field
- Gauss' law (integral form, application)
- Electric potential
- Capacitance and dielectrics, energy storage in capacitors
- Electric currents (dc circuits, energy, and power)
- Magnetic field (forces on moving charges and on current carrying wires)
- Ampere's law (integral form, applications, and the Biot-Savart law)
- Electromagnetic induction (Faraday's law in integral form)
- AC circuits (resistors, capacitors, inductors, resonance)
- Wave motion (electromagnetic, Poynting vector)
- Geometrical optics (refraction, lenses, mirrors, optical instruments)
- Physical optics (interference and diffraction, polarization)

TEXTBOOKS

Although the UT Austin Tests for Credit in Physics are not based on a specific textbook, the following books include material necessary to successfully complete the tests and would be appropriate for study and review.

Physics 303K and 303L

Raymond A. Serway, Physics for Scientists and Engineers, Philadelphia, Saunders College Publishing, 5th edition, 2000.

or

Douglas C. Giancoli, Physics for Scientists and Engineers, Prentice Hall College Div., 3rd edition, 2000.

Physics 301 and 316

Robert Resnick, David Halliday, and Jearl Walker, Extended, Fundamentals of Physics, 6th edition. New York: John Wiley & Sons, 2000.

This sheet contains most of the *basic* formulas in Physics 303K. For each of the problems encountered, students are encouraged to begin from a relevant subset of these formulas to arrive at the specific expressions needed. For this reason, many of the "ready-made" formulas given in the textbook have been systematically omitted. This sheet is intended to be a guide for doing homework problems, for reviewing for the exams, and to be a reference for Physics 303L and for your future use. **During any exam, questions on this sheet will not be answered. However, at other times your questions are very welcome.**

Mathematics (§2, §7, §11, Appendix B)

- Cartesian and polar coordinates: (§2)
  $x = r\cos\theta,\ y = r\sin\theta$
  $r^2 = x^2 + y^2,\ \tan\theta = \frac{y}{x}$
- Trig : $\cos\alpha\cos\beta + \sin\alpha\sin\beta = \cos(\alpha-\beta)$
  $\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$
  $\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$
  $\sin 2\theta = 2\sin\theta\cos\theta,\ \cos 2\theta = \cos^2\theta - \sin^2\theta$
- Vector algebra: $\vec{A} = (A_x, A_y) = A_x\hat{i} + A_y\hat{j}$
  Resultant: $\vec{R} = \vec{A} + \vec{B} = (A_x + B_x, A_y + B_y)$
  Method of parallelgram: R is diagonal.
- $\vec{A}\cdot\vec{B} = AB\cos\theta = A_xB_x + A_yB_y + A_zB_z$
- Cross product: $\hat{i}\times\hat{j} = \hat{k}, \hat{j}\times\hat{k} = \hat{i}, \hat{k}\times\hat{i} = \hat{j}$.

$$\vec{C} = \vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$C = AB\sin\theta,\ = A_\perp B = AB_\perp$, right hand rule.

Calculus: $\frac{dx^n}{dx} = nx^{n-1},\ \frac{d\ln x}{dx} = \frac{1}{x}$
$\frac{d\sin\theta}{d\theta} = \cos\theta,\ \frac{d\cos\theta}{d\theta} = -\sin\theta$

Measurements (§1)

- SI units; Standards of length, mass, time.
- Dimensional analysis: In any physics expression, dimension of each item must be the same.
  $e.g. [F] = [m][a] = MLT^{-2}, F = m^i v^j r^k$.
- Signif. figures: $c = ab, sf(c) = Min[sf(a), sf(b)]$
- Summation: $\sum_{i=1}^{N}(ax_i + b) = a\sum_{i=1}^{N} x_i + bN$

Motion(§3,4)

- One dim motion: $v = \frac{dx}{dt}, a = \frac{dv}{dt}$
  Average values: $\bar{v} = \frac{x_f - x_i}{t_f - t_i}, \bar{a} = \frac{v_f - v_i}{t_f - t_i}$
- One dim motion with constant acceleration:
  $vt:\ \ v = v_0 + at$
  $xt:\ \ x = \bar{v}t = v_0t + \frac{1}{2}at^2\ ,\ \bar{v} = \frac{v_0 + v}{2}$.
  $vx:\ \ v^2 = v_0^2 + 2ax$
- Nonunif. acc.: Method of separation of variables.
- The independent x and y motion
- Projectile motion: $t_{rise} = t_{fall} = \frac{t_{trip}}{2} = \frac{v_{0y}}{g}$
  $h = \frac{1}{2}gt_{fall}^2,\ R = v_{0x}t_{trip}$
  Circular: $a_c = \frac{v^2}{r}, v = \frac{2\pi r}{T}, f = \frac{1}{T}$ (Herts)

Curvilinear motion: $a = \sqrt{a_t^2 + a_r^2}$.

- Relative velocity: $\vec{v} = \vec{v}' + \vec{u}$

Law of Motion and applications (§5,6)

- Force: $\vec{F} = m\vec{a},\ W = mg,\ g = 9.8\ m/s^2$.
- Circular motion: $a_c = \frac{v^2}{r},\ v = \frac{2\pi r}{T} = 2\pi rf$
- Friction: $f_s \le \mu_s N, f_k = \mu_k N$
- Gravity: $F = \frac{Gm_1m_2}{d^2}$
- Equilibrium (concurrent forces): $\Sigma_i \vec{F}_i = 0$.

Energy (§7-8)

- Work (for all F) : $W_{A\to B} = Fs_{\parallel} = F_{\parallel}s = Fs\cos\theta$
  $= \vec{F}\cdot\vec{s} \to \int_A^B \vec{F}\cdot d\vec{s}$ (in Joules)
- Three types of effects due to work done:
  $F^{ext} = ma + F + f$
  $W^{ext}|_{A\to B} = (K_B - K_A) + (U_B - U_A) + W_f|_{A\to B}$
- Kinetic energy: $K_B - K_A = \int_A^B m\vec{a}\cdot d\vec{s},\ K = \frac{1}{2}mv^2$
- PE (conservative $\vec{F}$): $U_B - U_A = -\int_A^B \vec{F}\cdot d\vec{s}$
- $U_{gravity} = mgy,\ U_{spring} = \frac{1}{2}kx^2$
- From U to $\vec{F}$: $F_x = -\frac{\partial U}{\partial x},\ F_y = -\frac{\partial U}{\partial y}\ldots$
- $F_{gravity} = -\frac{\partial u}{\partial y} = -mg, F_{spring} = -\frac{\partial u}{\partial x} = -kx$.
- Power: $P = \frac{dW}{dt} = Fv_{\parallel} = Fv\cos\theta = \vec{F}\cdot\vec{v}$. (*watts*)

Collision (§9)

- Impulse : $\vec{I} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i \to \int_{t_i}^{t_f}\vec{F}dt$
- Momentum : $\vec{p} = m\vec{v}$
- Two-body,1 dim : $x_{cm} = \frac{m_1x_1 + m_2x_2}{M}, M = m_1 + m_2$
  $P_{cm} \equiv Mv_{cm} = p_1 + p_2$
  $F_{cm} \equiv F_1 + F_2 = m_1a_1 + m_2a_2 = Ma_{cm}$
  $K_1 + K_2 = K_1^* + K_2^* + K_{cm}$
- Two-body collision: $\vec{P}_i = \vec{P}_f = (m_1 + m_2)\vec{v}_{cm}$
  $v_i^* = v_i - v_{cm}, v_i' = v_i^{*\prime} + v_{cm}$
- Elastic: $v_i^{*\prime} = -v_i^*, v_i' = 2v_{cm} - v_i$
- Many body, center of mass:
  $\vec{r}_{cm} = \frac{\Sigma_i m_i\vec{r}_i}{M} = \frac{\int \vec{r}dm}{M},\ M = \Sigma_i m_i$
- Force on cm : $\vec{F}^{ext} = \frac{d\vec{P}}{dt} = M\vec{a}_{cm},\ \vec{P} = \Sigma_i\vec{p}_i$

Rotation of Rigid-Body (§10)

- Kinematics: $\theta = \frac{s}{r},\ \omega = \frac{v}{r},\ \alpha = \frac{a_t}{r}$.
- Moment of inertia: $I = \Sigma_i m_i r_i^2 = \int r^2 dm$
  $I_{disk} = \frac{1}{2}MR^2, I_{ring} = \frac{1}{2}M(R_1^2 + R_2^2)$
  $I_{rod} = \frac{1}{12}M\ell^2,\ I_{rectangle} = \frac{1}{12}M(a^2 + b^2)$
  $I_{sphere} = \frac{2}{5}MR^2,\ I_{sph-shell} = \frac{2}{3}MR^2$
  $I = M *$ (Radius of gyration)^2
  $I = I_{cm} + MD^2$

- Kinetic energies: $K_{rot} = \frac{1}{2} I\omega^2$, $K = K_{rot} + K_{cm}$
- Ang.momentum: $L = rmv \rightarrow \sum_i r_i m_i(\omega r_i) = I\omega$
- Torque: $\tau = \frac{dL}{dt} = m\frac{dv}{dt}r = Fr \rightarrow I\frac{d\omega}{dt} = I\alpha$
- $W^{ext} = \Delta K + \Delta U + W_f$, $K = K_{rot} + \frac{1}{2}mv^2$, $P = \tau\omega$

Rolling, angular momentum and torque (§11)

- Rolling: $K = \frac{1}{2}(I_c + MR^2)\omega^2 = \frac{1}{2}(\frac{I_c}{R^2} + M)v^2$
- Angular momentum: $\vec{L} = \vec{r} \times \vec{p}$, $L = r_\perp p \rightarrow I\omega$.
- Torque: $\vec{\tau} = \frac{d\vec{L}}{dt} = \vec{r} \times \frac{d\vec{p}}{dt} = \vec{r} \times \vec{F}$, $\tau = r_\perp F \rightarrow I\alpha$.
- Gyroscope: $\omega_p = \frac{d\phi}{dt} = \frac{1}{L}\frac{dL}{dt} = \frac{\tau}{L} = \frac{mgh}{I\omega}$

Static equilibrium (§12)

- $\Sigma \vec{F}_i = 0$, about any point $\Sigma \vec{\tau}_i = 0$.
- Subdivisions: $\vec{r}_{cm} = \frac{m_A \vec{r}_{Acm} + m_B \vec{r}_{Bcm}}{m_A + m_B}$

Oscillation motion (§13) $f = \frac{1}{T}$, $\omega = \frac{2\pi}{T}$, $\lambda = vT$.

- SHM : $a = \frac{d^2x}{dt^2} = -\omega^2 x$, $\alpha = \frac{d^2\theta}{dt^2} = -\omega^2\theta$
  $x = x_{max}\cos(\omega t + \delta)$, $x_{max} = A$
  $v = -v_{max}\sin(\omega t + \delta)$, $v_{max} = \omega A$
  $a = -a_{max}\cos(\omega t + \delta) = -\omega^2 x$, $a_{max} = \omega^2 A$
- $E = K + U = K_{max} = \frac{1}{2}m(\omega A)^2 = U_{max} = \frac{1}{2}kA^2$.
- Spring : $ma = -kx$
- Simple pendulum : $a_t = \alpha\ell = -g\sin\theta$
- Physical pendulum: $\tau = I\alpha = -mgd\sin\theta$
- Torsion pendulum : $\tau = I\alpha = -\kappa\theta$.

Gravity (§14)

- At $r \geq R$: $g_r = \frac{GM}{r^2} = g\left(\frac{R}{r}\right)^2$
- Orbit: $a_c = \frac{v^2}{r} = \omega^2 r = \left(\frac{2\pi}{T}\right)^2 r = g_r$
  $U = -\frac{GmM}{r}$, $F = -\frac{dU}{dr} = -\frac{GmM}{r^2}$,
  $G = 6.67 \times 10^{-11} Nm^2/kg^2$, $R_{earth} = 6370$ km .
- Kepler's Laws of planetary motion

Elliptical orbit: Sun at one focal point.

$L = rm\frac{\Delta r_\perp}{\Delta t} \rightarrow \frac{\Delta A}{\Delta t} = \frac{1}{2}\frac{r\Delta r_\perp}{\Delta t} = \text{const}$

$\frac{GM}{r^2} = \left(\frac{2\pi r}{T}\right)^2 \frac{1}{r} \rightarrow T^2 = \frac{4\pi^2}{GM}a^3$. ($a$: semimajor axis)

Fluid mechanics (§15)

- Pascal: $P = \frac{F_1}{A_1} = \frac{F_2}{A_2}$, $1atm = 1.013 \times 10^5 N/m^2$.
- Archimedes: $B = Mg$. Pascal $= N/m^2$.
- $P = P_{atm} + \rho gh$, with $P = \frac{F}{A}$ and $\rho = \frac{m}{V}$.
- $F = \int PdA \rightarrow \rho g\ell \int_0^h (h-y)dy$.
- Continuity: Av=constant
- Bernoulli: $P + \frac{1}{2}\rho v^2 + \rho gy$=const.

Wave motion (§16)

- Traveling waves: $y = f(x - vt)$, $y = f(x + vt)$
  Right moving: $y = A\sin(kx - \omega t - \phi)$
- Along a string: $v = \sqrt{\frac{F}{\mu}}$
- General: $\Delta E = \Delta K + \Delta U = \Delta K_{max}$
  $P = \frac{\Delta E}{\Delta t} = \frac{1}{2}\frac{\Delta m}{\Delta t}(\omega A)^2$
- 1 dim waves: $\frac{\Delta m}{\Delta t} = \frac{\Delta m}{\Delta x} \cdot \frac{\Delta x}{\Delta t} = \frac{\Delta m}{\Delta x} \cdot v$
- †Circular: $\frac{\Delta m}{\Delta t} = \frac{\Delta m}{\Delta A} \cdot \frac{\Delta A}{\Delta r} \cdot \frac{\Delta r}{\Delta t} = \frac{\Delta m}{\Delta A} \cdot 2\pi r v$
- †Spherical: $\frac{\Delta m}{\Delta t} = \frac{\Delta m}{\Delta V} \cdot 4\pi r^2 v$

Sound (§17)

$v = \sqrt{\frac{B}{\rho}}$, $s = s_{max}\cos(kx - \omega t - \phi)$

$\Delta P = -B\frac{\Delta V}{V} = -B\frac{\partial s}{\partial x}$

$\Delta P_{max} = Bks_{max} = \rho v\omega s_{max}$

- Piston: $\frac{\Delta m}{\Delta t} = \frac{\Delta m}{\Delta V} \cdot \frac{A\Delta x}{\Delta t} = \rho Av$
- Intensity: $I = \frac{P}{A} = \frac{1}{2}\rho v(\omega s_{max})^2$
- Intensity level: $\beta = 10\log_{10}\frac{I}{I_0}$, $I_0 = 10^{-12}\frac{W}{m^2}$
- Plane waves: $\psi(x,t) = c\sin(kx - \omega t)$
- †Circular waves: $\psi(r,t) = \frac{c}{\sqrt{r}}\sin(kr - \omega t)$
- †Spherical: $\psi(r,t) = \frac{c}{r}\sin(kr - \omega t)$
- Doppler effect: $\lambda = vT$, $f_0 = \frac{1}{T}$, $f' = \frac{v'}{\lambda'}$.
  Example: $\vec{v}_e = \vec{v}_{eg} > 0$, $\vec{v}_d = \vec{v}_{dg} < 0$, $\vec{v}_{wind} > 0$.
  Modified wave: $\lambda' = \lambda - v_{emitter}T + v_{wind}T$
  Detected wave speed: $v' = v + v_{wind} + v_{detector}$
  $f' = \frac{v'}{\lambda'} = \frac{v_{sound} + v_{wind} + v_{detector}}{v_{sound} + v_{wind} - v_{emitter}} f_0$
- Received and reflected frequencies are the same.
- Shock waves: Mach Number$= \frac{v_{source}}{v_{sound}} = \frac{1}{\sin\theta}$

Superposition of waves (§18)

- Phase difference: $\sin(kx - \omega t) + \sin(kx - \omega t - \phi)$
- Standing waves: $\sin(kx - \omega t) + \sin(kx + \omega t)$
- Beats: $\sin(kx - \omega_1 t) + \sin(kx - \omega_2 t)$
- †Others: $a\cos(kx - \omega t) + b\sin(kx - \omega t)$.
  $y = \sin(kx - \omega t)$, $z = \sin(kx - \omega t)$.
- Fundamental modes: Sketch wave patterns.
  String: $\frac{\lambda}{2} = \ell$.
  Open-open pipe: $\frac{\lambda}{2} = \ell$.
  Open-closed pipe: $\frac{\lambda}{4} = \ell$.
  Rod clamped middle: $\frac{\lambda}{2} = \ell$.
- n antinodes on string: $n\frac{\lambda}{2} = \ell$

Physics 303L Course Summary (Chiu 4/27/94)

§23 Electric force and electric field

Electric force between 2 point charges:

- Coulomb: $|F| = \frac{k|q_1 q_2|}{r^2}$, $k = 9.0 \times 10^9 \frac{Nm^2}{C^2}$
  $\epsilon_0 = \frac{1}{4\pi k} = 8.85 \times 10^{-12} \frac{C^2}{Nm^2}$
- $e = 1.6 \times 10^{-19} C$, $m_e = 9.11 \times 10^{-31} kg$
- $e_p = +e$, $m_p = 1.67 \times 10^{-27} kg$

Electric field: $\vec{E} = \frac{\vec{F}}{q}$

- Point charge: $|E| = \frac{k|Q|}{r^2}$, $\vec{E} = \vec{E}_1 + \vec{E}_2 + \ldots$
- Field patterns: point charge, dipole, $\|$ plates, rod, spheres, cylinders...
- Charge distributions:
  Linear charge density: $\lambda = \frac{\Delta Q}{\Delta x}$
  Area charge density: $\sigma_A = \frac{\Delta Q}{\Delta A}$
  Surface charge density: $\sigma_{surf.} = \frac{\Delta Q_{surf}}{\Delta A}$
  Volume charge density: $\rho = \frac{\Delta Q}{\Delta V}$

§24 Electric flux and Gauss' law

- Flux: $\Delta\Phi = E\Delta A_\perp = \vec{E} \cdot \hat{n}\Delta A$
- Gauss law: Outgoing flux from S, $\Phi_S = \frac{Q_{enclosed}}{\epsilon_0}$
  Steps: Inspect $\vec{E}$ pattern, construct S.
  Find $\Phi_s = \oint_S \vec{E} \cdot d\vec{A}$, $Q_{encl.}$ Solve $\vec{E}$.
- Spherical: $\Phi_S = 4\pi r^2 E$
  Cylindrical: $\Phi_S = 2\pi r \ell E$
  Pill box: $\vec{E}$ at 1 side, $\Phi_S = E\Delta A$
  $\vec{E}$ at 2 sides, $\Phi_S = 2E\Delta A$
  Conductor: $\vec{E}_{in} = 0$, $E^{\|}_{surf} = 0$, $E^{\perp}_{surf} = \frac{\sigma_{surf}}{\epsilon_0}$.

§25 Potential

- Potential energy: $\Delta U = q\Delta V$, $1eV = 1.6 \times 10^{-19} J$
- Positive charge moves from high V to low V
- Point charge: $V = \frac{kQ}{r}$, $V = V_1 + V_2 = \ldots$
- Energy of a charge-pair: $U = \frac{kq_1 q_2}{r_{12}}$
- Potential difference: $|\Delta V| = |\vec{E}\Delta s_{\|}|$,
  $\Delta V = -\vec{E} \cdot \Delta\vec{s}$, $V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{s}$
  $E = -\frac{dV}{dr}$; $E_x = -\frac{\Delta V}{\Delta x}|_{fix\ y,z} = -\frac{\partial V}{\partial x}$, *etc.*

§26 Capacitances: $Q = CV$

- Series: $V = \frac{Q}{C} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \ldots$
  with $Q = Q_1 = Q_2 = \ldots$, $V = V_1 + V_2 + \ldots$
- Parallel: $Q = CV = C_1V_1 + C_2V_2 + \ldots$
  with $Q = Q_1 + Q_2 + \ldots$, $V = V_1 = V_2 = \ldots$
- Parallel plate-capacitor: $C = \frac{Q}{V} = \frac{Q}{Ed} = \frac{\epsilon_0 A}{d}$
- Energy: $U = \int_0^Q V dq = \frac{1}{2} \cdot \frac{Q^2}{C}$, $u = \frac{1}{2}\epsilon_0 E^2$.
- Dielectrics $C = \kappa C_0$, $U_\kappa = \frac{1}{2\kappa}\frac{Q^2}{C_0}$, $u_\kappa = \frac{1}{2}\epsilon_0 \kappa E_\kappa^2$
- Spherical capacitor: $V = \frac{Q}{4\pi\epsilon_0 r_1} - \frac{Q}{4\pi\epsilon_0 r_2}$

§27 Current and resistance

- Current: $I = dQ/dt = nqv_dA$
  $n =$ (valence − el/atm) $*$ N_A/molar-volume
  density ρ=molar-mass/molar-volume
- Ohm's law: $V = IR$, $E = \rho J$
  $E = \frac{V}{l}$, $J = \frac{I}{A}$, $R = \frac{\rho\ell}{A}$
- Power: $P = IV = \frac{V^2}{R} = I^2R$
- Thermal coefficient of ρ: $\alpha = \frac{\Delta\rho}{\rho_0 \Delta T}$
- Motion of free electrons in an ideal conductor:
  $a\tau = v_d \rightarrow \frac{qE}{m}\tau = \frac{J}{nq} \rightarrow \rho = \frac{m}{nq^2\tau}$.

§28 Direct current circuits

- Series: $V = IR = I_1R_1 + I_2R_2 + \ldots$
  where $I = I_1 = I_2 = \ldots$, $V = V_1 + V_2 + \ldots$
- Parallel: $I = \frac{V}{R} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \ldots$
  where $I = I_1 + I_2 \ldots$, $V = V_1 = V_2 = \ldots$
- Steps in application of Kirchhoff's Rules
  –Label currents
  –Node equations: $\Sigma i_{in} = \Sigma i_{out}$
  –Loop equations: "$\Sigma(\pm\mathcal{E}) + \Sigma(\mp iR)=0$"
  Natural:"+" for loop-arrow entering − terminal
  "−" for loop-arrow-parallel to current flow
- Thm: If $\frac{dy}{dt} = -ay$, $y = y_0$ at $t = 0 \rightarrow y = y_0 e^{-at}$.
  Charging:$\mathcal{E} - \frac{q}{C} - Ri = 0$, $\frac{1}{C}\frac{dq}{dt} + R\frac{di}{dt} = \frac{i}{C} + R\frac{di}{dt} = 0$
  Discharge: $0 = V_C - Ri = \frac{q}{C} + R\frac{dq}{dt}$, $\frac{i}{C} + R\frac{di}{dt} = 0$
  Exponential law: $i = i_0 \exp(-\frac{t}{RC})$

§29 Magnetic fields

- Long wire: $B = \frac{\mu_0 i}{2\pi r}$, $(rhr\#1)$, $\mu_0 = 4\pi \times 10^{-7}$
- Magnetic force: $\vec{F}_M = i\vec{\ell} \times \vec{B} \rightarrow q\vec{v} \times \vec{B}$
- Orbit: $Bqv = \frac{mv^2}{r}$, $\frac{Bq}{m} = \frac{v}{r} = \omega = 2\pi f = \frac{2\pi}{T}$.
- Cross product: $\vec{C} = \vec{A} \times \vec{B}$, $(rhr\#2)$; $\hat{i} \times \hat{j} = \hat{k}$.
- Loop-magnet ID: $\vec{\tau} = i\vec{A} \times \vec{B}$, $\vec{\mu} = iA\hat{n}$.$(rhr\#3)$
- Hall effect: $V_H = Fd/q = Bvd$.

§30 Sources of Magnetic Field

- Ampere's law: $\mathcal{M} = \oint \vec{B} \cdot d\vec{s} = \mu_0 I_{encircled}$
  Steps: Inspect $\vec{B}$-pattern, construct $\mathcal{L}$.
  Find $\mathcal{M}$, $I_{enc.}$ Solve for $\vec{B}$.
  Superposition principle.
- Biot-Savart Law: $\Delta\vec{B} = \frac{\mu_0}{4\pi}\frac{i\Delta\vec{\ell} \times \hat{r}}{r^2} \rightarrow B = \frac{\mu_0}{4\pi}\frac{q\vec{v} \times \hat{r}}{r^2}$.
  Long wire: $\Delta B = \frac{\mu_0}{4\pi}\frac{i\Delta y \sin\theta}{r^2}$, $\sin\theta = \frac{a}{r}$, $\frac{\Delta y}{\Delta\theta} = \frac{r^2}{a}$
  Circular arc: $\Delta B = \frac{\mu_0}{4\pi}\frac{ir\Delta\theta}{r^2}\sin 90° = \frac{\mu_0 i\Delta\theta}{4\pi r}$.
- Displ. current: $I_d = \epsilon_0\frac{d\Phi_E}{dt} = \epsilon_0\frac{dEA}{dt} = \frac{dQ}{dt}$
- Magnetism in atom
  Orbital motion: $\mu = iA = \frac{e}{2m}L$
  $L = mvr = n\hbar$, $\hbar = \frac{h}{2\pi} = 1.06 \times 10^{-34} Js$.
  $\mu_{orbit} = n\mu_B$, $\mu_B = \frac{e\hbar}{2m} = 9.27 \times 10^{-24} J/T$.
  Spin: $S = \frac{\hbar}{2}$, $\mu_{spin} = \mu_B$.
- Magnetism in matter: $B = B_0 + B_M = (1 + \chi)B_0$
  $B = \kappa_m H$, $\kappa_m = 1 + \chi$, $H = B_0/\mu_0$.
  Ferromagnetic: $\chi >> 1$, Hysteresis loop.
  Paramagnetic: $0 < \chi << 1$, $M = \frac{C}{T}B$.
  Diamagnetic: $-1 << \chi < 0$.

§31 Faraday's law

- $\mathcal{E} = -N\frac{d\phi_B}{dt}$, $\phi_B = \int \vec{B} \cdot d\vec{A}$
  $\mathcal{E} = \oint \vec{E} \cdot d\vec{s}$, $\vec{E} = \frac{\vec{F}_M}{q}$. Cir. loop: $\mathcal{E} = 2\pi r E$.
- Lenz law: Induced $\vec{B}_{ind}$ opposes change of Φ_B.

- $\frac{d\Phi_E}{dt} = \frac{dBA}{dt} = \frac{dB}{dt}A_{\perp} + B\frac{dA}{dt}$.
- Moving rods: $\frac{dA}{dt} = \ell v$. $\frac{dA}{dt} = \frac{d}{dt}(\frac{1}{2}R \cdot R\theta) = \frac{R^2\omega}{2}$.
- Rotating loop: $\frac{dA}{dt} = \frac{d}{dt}(A\cos\omega t)$, $\omega = 2\pi f$.
- Cutting B lines — $E_{ind} = \frac{F}{q} = \vec{v} \times \vec{B}$ — $\mathcal{E}_{ind}$
- Maxwell equations:

  $\oint \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0}$. $\oint \vec{B} \cdot d\vec{A} = 0$

  $\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$. $\oint \vec{B} \cdot d\vec{s} = \mu_0[I + \epsilon_0\frac{d\Phi_E}{dt}]$
- Lorentz force: $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$

§32 **Inductance**

- Mutual: $\mathcal{E}_2 = -M_{21}\frac{di_1}{dt}$, $M_{21} = M_{12} = \frac{N_2\Phi_{21}}{i_1}$.
- Self: $\mathcal{E} = -L\frac{di}{dt}$, $L = \frac{N\Phi}{i}$, $V_L = L\frac{di}{dt}$
- Solenoid: $L = NBA/i$, $B = \mu_0 ni$.
- Energies: $U_L = \frac{1}{2}Li^2$, $u_B = \frac{1}{2\mu_0}B^2$

  $U_C = \frac{1}{2C}q^2$, $u_E = \frac{1}{2}\epsilon_0 E^2$
- LC: $V_L = V_C \Rightarrow L\frac{di}{dt} = -L\frac{d^2q}{dt^2} = \frac{q}{C}$

  $q = q_0\cos(\omega t + \delta)$, $\omega = \sqrt{\frac{1}{LC}}$, $\omega = 2\pi f$.

  $U_C + U_L = U_{C\,max} = U_{L\,max} = U_0$.
- Theorem: If $\frac{dy}{dt} = -ay$, $y = y_0\exp(-at)$.
- LR: $\mathcal{E} = V_L + Ri$, $\frac{dV_L}{dt} + \frac{RV_L}{L} = 0$

  $V_L = \epsilon\exp\left(-\frac{Rt}{L}\right) \rightarrow i = \frac{\epsilon}{R}\left[1 - \exp\left(\frac{-Rt}{L}\right)\right]$
- LRC: $Q \approx Q_0 e^{-\frac{R}{2L}t}\cos\omega_d t$, $\omega_d = \left[\frac{1}{LC} - \left(\frac{R}{2L}\right)^2\right]^{\frac{1}{2}}$

  Damping: Under-, critically and over-damped.

§33 **AC circuits** Mean: $\bar{f}(t) = \left[\int_0^T f(t)dt\right]/T$.

$\overline{\text{constant}} = \text{constant}$, $\overline{\sin\omega t} = \overline{\cos\omega t} = 0$,

$\overline{\sin 2\omega t} = \overline{\cos 2\omega t} = \overline{\sin\omega t\cos\omega t} = 0$

$|\sin\omega t|_{rms} = \overline{|\sin^2\omega t|}^{\frac{1}{2}} = \overline{\left[\frac{1}{2}(1 - \cos 2\omega t)\right]}^{\frac{1}{2}} = \frac{1}{\sqrt{2}}$.

§34 **Electromagnetic waves**

- Properties of em waves:

  $E = E_m\cos(kz - \omega t)$, $B = \frac{E}{c}$

  $v = \frac{dz}{dt} = \frac{\omega}{k} = \lambda f = \frac{\lambda}{T}$, $n = \frac{c}{v}$.

  speed of light: $c = \frac{1}{\sqrt{\epsilon_0\mu_0}} = 3 \times 10^8$ m/sec

  $\vec{B} \perp \vec{E}$, propagating along: $\vec{E} \times \vec{B}$

  $u = u_E + u_B$, $u_E = u_B$
- Poynting vector: $\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0}$, $\bar{S} = \bar{I} = \frac{E_{rms}B_{rms}}{\mu_0}$.

  Intensity: $I = \frac{Power}{A} = \frac{\Delta U}{A\Delta z}\frac{dz}{dt} = uc$
- Energy conservation: $\int \vec{S} \cdot d\vec{A} = \frac{dU}{dt} + P_R$
- Complete absorption: Momentum $p = \frac{U}{c}$

  Pressure= $\frac{F}{A} = \frac{\Delta p}{\Delta t} \cdot \frac{1}{A} = \frac{\Delta U}{c\Delta t} \cdot \frac{1}{A} = u = \frac{S}{c}$
- Complete reflection: $p = \frac{2U}{c}$, Pressure= $\frac{2S}{c}$.
- General case: $P = \left[(1 - \eta)\frac{S}{c} + \eta\frac{2S}{c}\right]\cos\theta$
- Doppler: Emitter-detector approaching each other:

  Modified wave: $\lambda' = \lambda - v_{emit}T = \frac{c}{f_0} - \frac{v_{emit}}{f_0}$

  Detected wave front speed: $v' = c + v_{detector}$

  $f' = \frac{v'}{\lambda'} = \frac{c + v_{detector}}{c - v_{emitter}} f_0$

§35-36 **Geometric optics**

- **Reflection and Refraction**
- Index of refraction: $\frac{n_1}{n_2} = \frac{v_2}{v_1} = \frac{\lambda_2}{\lambda_1}$.
- Snell's law: $n_1\sin\theta_1 = n_2\sin\theta_2$
- Critical angle: $n_2 > n_1$, $n_2\sin\theta_c = n_1\sin 90°$
- Total reflection: $\theta > \theta_c$
- **Mirrors and lenses**: $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$
- Ray tracing rules.

  Mirror: At S, reflected symmetrically.
  Through center, undeflected.
  Parallel to axis, converges toward F (or diverges away from F), $f = \frac{R}{2}$.

  Lens: Through center, undeflected
  Parallel to axis, converges toward F (or diverges away from F).
- Image: $s' > 0$ (real), $s' < 0$ (virtual)
- Focal point F: at $s = \infty$, $s' = f$

  $f = \pm|f|$, "+" convergent, "−" divergent.
- Magnification: $M = \frac{h'}{h} = -\frac{s'}{s}$
- Refraction at spherical surface: $\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R}$.

  R is coordinate of center with origin at S, with S the symmetry point of surface on the axis.
- Lens maker: $\frac{1}{f} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$
- Huygen's principles:

  Points in wave front are sources of next wavelets.
  Forward tangent surface is next wave front.
- **Wave front method**:

  Optical curvature: $C = \frac{n}{r}$, $r = \pm|r|$.

  $C_{after} = C_{before} + C_0$

  Lens: $C_0 = \frac{n}{f}$, $f = \pm|f|$

  Spherical surface: $C_0 = \frac{n_{after} - n_{before}}{R}$, $R = \pm|R|$
- Telescope: $\theta_{eye} = -h'/f_e$, $\theta_{object} = h'/f_o$

§37 **Interference**

- Maxima $\phi = 0, 2\pi, 4\pi \ldots$; minima $\phi = \pi, 3\pi, 5\pi \ldots$
- Double slits: $I_{av} = I_0\cos^2\left(\frac{\phi}{2}\right)$, $\phi = k\Delta$

  $\sin\theta = \frac{\Delta}{d}$, $\tan\theta = \frac{y}{L}$; Small θ: $\theta \approx \sin\theta \approx \tan\theta$.
- Phasor diagram: $\vec{A} = \vec{A}_1 + \vec{A}_2 + \vec{A}_3 + \ldots$

  $A_x = A_{1x} + A_{2x} \ldots$, $A_y = A_{1y} + A_{2y} + \ldots$

  $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$, $c^2 = a^2 + b^2 - 2ab\cos\gamma$.
- First minimum for N slits: $\phi = \frac{2\pi}{N}$
- Thin film: $\phi = k\Delta + |\phi_{refl1} - \phi_{refl2}|$, $\Delta = 2t$

  $\phi_{refl} = \pi$, (by denser med.), 0 (by lighter med).

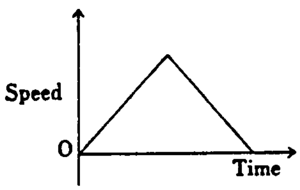
§38 **Diffraction and polarization**:

- Single slit: $I = I_0\left[\frac{\sin\frac{\beta}{2}}{\frac{\beta}{2}}\right]^2$, $\beta = k\Delta$, $\Delta = a\sin\theta$
- Resolution criterion: $\theta_m = 1.22\lambda/D$
- Grating: Principle maxima. $\Delta = m\lambda$
- Brewster $(n_1 < n_2)$: $n_1\sin\theta_1 = n_2\sin(\frac{\pi}{2} - \theta_1)$
- Polarizer: $E_{transmit} = E_0\cos\theta$, $I = I_0\cos^2\theta$
- Unpolarized light:

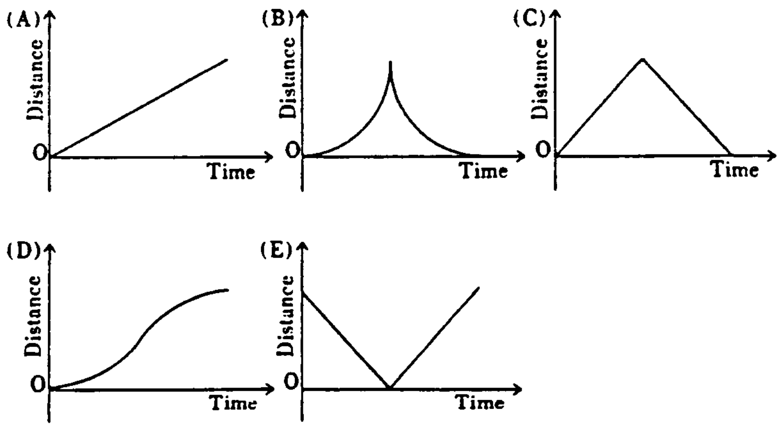
  Take any set of mutually perpendicular axes, 50% along one axis, 50% along the other axis.

Part I—Mechanics

MULTIPLE-CHOICE QUESTIONS

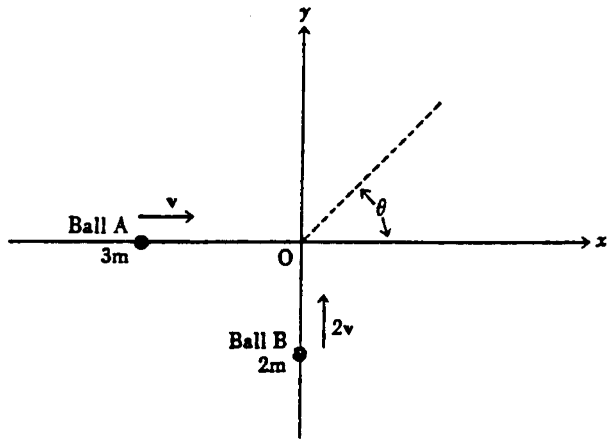


1. An automobile makes a short trip along a straight line. The speed of the automobile as a function of time is shown by the graph above. Which of the following graphs best represents the distance traveled by the automobile as a function of time?



Questions 2–3

Two small balls of clay move over a horizontal frictionless surface, collide, stick together, and continue moving. Their initial directions are shown in the diagram below. The particles collide at the origin O. The mass of ball A is 3m and its speed is v. The mass of ball B is 2m and its speed is 2v.

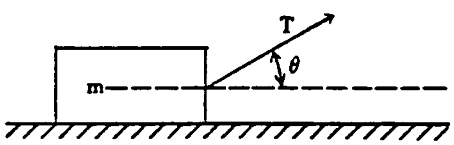


2. After the collision, the balls move along the dashed line at an angle θ with the x-axis. The value of tangent θ is

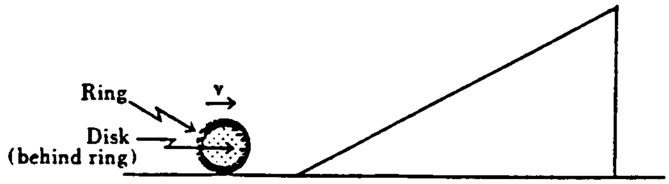
(A) $\frac{2}{3}$ (B) $\frac{3}{4}$ (C) $\frac{4}{3}$ (D) 2 (E) $\frac{8}{3}$

3. After the collision, A and B move together as a unit at a speed of

(A) $\frac{1}{5}$v (B) v (C) $\frac{7}{5}$v (D) $\sqrt{5}$ v (E) 3v



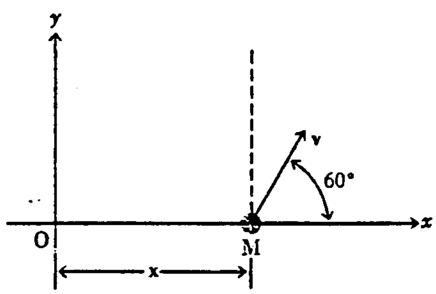
4. A block of mass m pulled with constant velocity over a floor by a force T inclined at an angle θ with the floor as shown above. The coefficient of friction between the block and floor is μ. The magnitude of the frictional force is
(A) $T\cos\theta$
(B) $T\sin\theta$
(C) 0
(D) μmg
(E) $\mu T\cos\theta$



5. A ring and a disk have identical masses, radii, and velocities. If the ring and the disk roll without slipping up an inclined plane, how will the distances that the ring and disk move up the plane before coming to rest compare?
   (A) The ring will move farther than the disk.
   (B) The disk will move farther than the ring.
   (C) The ring and the disk will move equal distances.
   (D) The relative distances depend on the angle of elevation of the plane.
   (E) The relative distances depend on the length of the plane.

6. A particle of mass m and speed v collides at right angles with a very massive wall in a perfectly elastic collision. The magnitude of the change of momentum of the particle is
   (A) zero (B) mv/2 (C) mv (D) $\sqrt{2}$mv (E) 2mv

7. The momentum of a particle in kilogram·meter/second depends on time according to the equation $p = 2t^2 + 4$ where t is in seconds. At the end of 1 second the force on the particle is
   (A) 2.0 newtons
   (B) 4.0 newtons
   (C) 4.7 newtons
   (D) 6.0 newtons
   (E) 8.0 newtons

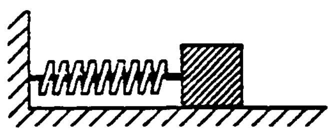


8. At the instant a particle of mass M crosses the x-axis, it is moving with a speed v and at an angle of 60° to the x-axis, as shown above. The angular momentum of the particle about an axis through the origin O and perpendicular to the page at this instant is

(A) Mvx directed out of the page
(B) Mvx directed in the +y direction
(C) Mvx directed in the +x direction
(D) $\frac{Mvx}{2}$ directed out of the page
(E) $\frac{\sqrt{3}\,Mvx}{2}$ directed out of the page

9. Two objects having masses of m_1 and m_2 are a distance R apart in an inertial frame of reference. They are isolated from all forces except the gravitational force between them. As a result of this gravitational force, the objects experience accelerations of a_1 and a_2, respectively. When the distance has been reduced to R/2, which of the following is true?

(A) a_1 must be four times a_2.
(B) a_1 must be equal to a_2.
(C) a_1 and a_2 must each have been doubled.
(D) a_1 and a_2 must each have been multipled by four.
(E) a_1 and a_2 are unchanged.

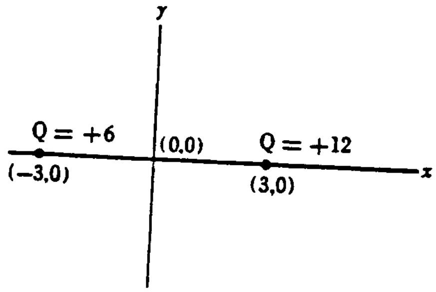


10. A block attached to a spring of negligible mass undergoes simple harmonic motion on a frictionless surface. The potential energy of the system is zero at the equilibrium position and has a maximum value of 50 joules. When the displacement of the block is half the maximum value, its instantaneous kinetic energy is

(A) 0 joules
(B) 12.5 joules
(C) 25 joules
(D) 37.5 joules
(E) 50 joules

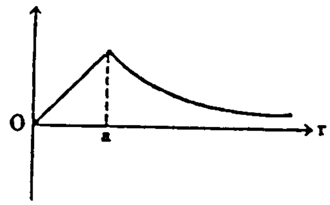
Sample Questions – Part II - Electricity and Magnetism

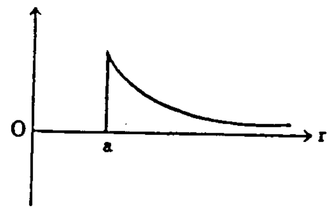
MULTIPLE-CHOICE QUESTIONS

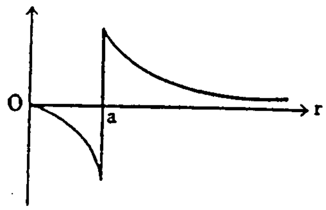


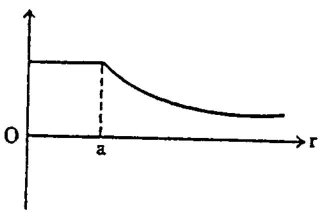
1. A charge of + 12 units is located in the xy-plane of a coordinate system at (+ 3, 0) and a second charge of + 6 units is located at (− 3, 0) as shown above. Where on the x-axis should an additional charge of + 24 units be located to produce an electric field equal to zero at the origin (0, 0)?
   (A) $x = -6$ (B) $x = +6$ (C) $x = -2$
   (D) $x = +2$ (E) $x = +1$

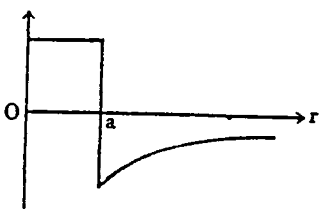
2. The two plates of a parallel-plate capacitor are a distance d apart and are mounted on insulating supports. A battery is connected across the capacitor to charge it and is then disconnected. The distance between the insulated plates is then increased to 2d. If fringing of the field is still negligible, which of the following quantities is doubled?
   (A) The capacitance of the capacitor
   (B) The total charge on the capacitor
   (C) The surface density of the charge on the plates of the capacitor
   (D) The energy stored in the capacitor
   (E) The intensity of the electric field between the plates of the capacitor

(A)


(B)


(C)


(D)


(E)


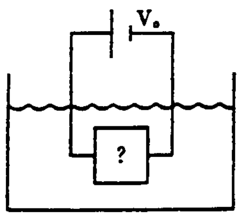
3. Which graph best represents the electric field inside and outside of a positively charged conducting shell with a radius a as a function of the distance r from the center of the shell?
(A) A (B) B (C) C (D) D (E) E

4. Which graph best represents the electric potential inside and outside the shell, in question 3, as a function of the distance r from the center of the shell?
(A) A (B) B (C) C (D) D (E) E

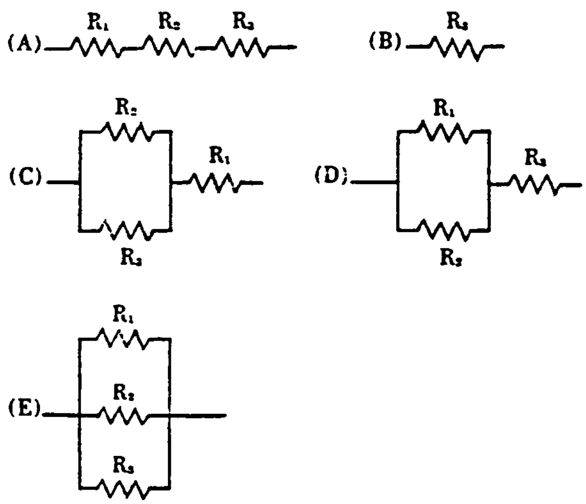
5. A proton has a mass of 1 atomic mass unit and a charge of + e, and an alpha particle has a mass of 4 atomic mass units and a charge of + 2e. A proton and an alpha particle are accelerated from rest through a potential difference of 100 volts.
What is the value of the ratio

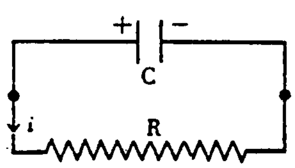
$$\frac{\text{resulting energy of the proton}}{\text{resulting energy of the alpha particle}}?$$

(A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{2}}$ (C) 1 (D) $\sqrt{2}$ (E) 2



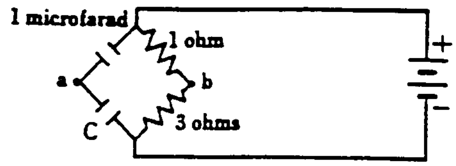
6. Suppose you are given a constant voltage source V_0 and three resistors R_1, R_2, and R_3 with $R_1 > R_2 > R_3$. If you wish to heat water in a pail, which of the following combinations of resistors will give the most rapid heating?



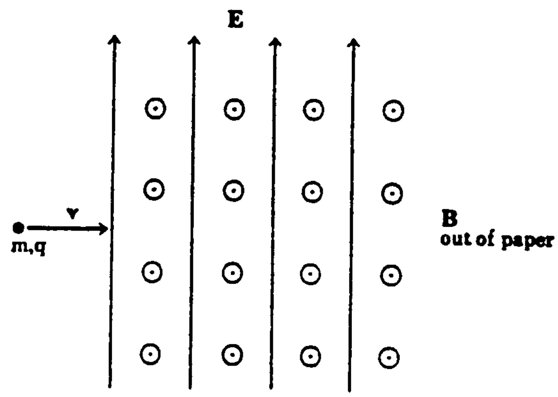


7. The terminals of a charged capacitor with capacitance C are connected to the terminals of a resistor with resistance R and the capacitor is thereby discharged. The time required for the capacitor to lose one-half its charge depends only on the

(A) original charge on the capacitor
(B) original potential difference between the terminals of the capacitor
(C) capacitance C of the capacitor
(D) resistance R of the resistor



8. In the circuit shown above, the potential difference between points a and b is zero for a value of capacitance C of
   (A) 1/3 microfarad
   (B) 2/3 microfarad
   (C) 2 microfarads
   (D) 3 microfarads
   (E) 9 microfarads



9. A particle of mass m and charge q moves with constant velocity **v** in a region of crossed electric and magnetic fields as shown above. Suppose the magnitude of **B** is now doubled. The particle will remain undeflected if
   (A) **E** is changed to half its former value
   (B) **E** is doubled
   (C) **v** is doubled
   (D) both **E** and **v** are doubled
   (E) **E** is doubled and **v** is halved

Answer Key Parts I and II

C EXAMINATION

Part I, Mechanics

1—D, 2—C, 3—B, 4—A, 5—A, 6—E, 7—B, 8—E, 9—D, 10—D

Part II, Electricity and Magnetism

1—A, 2—D, 3—B, 4—D, 5—A, 6—E, 7—E, 8—A, 9—B