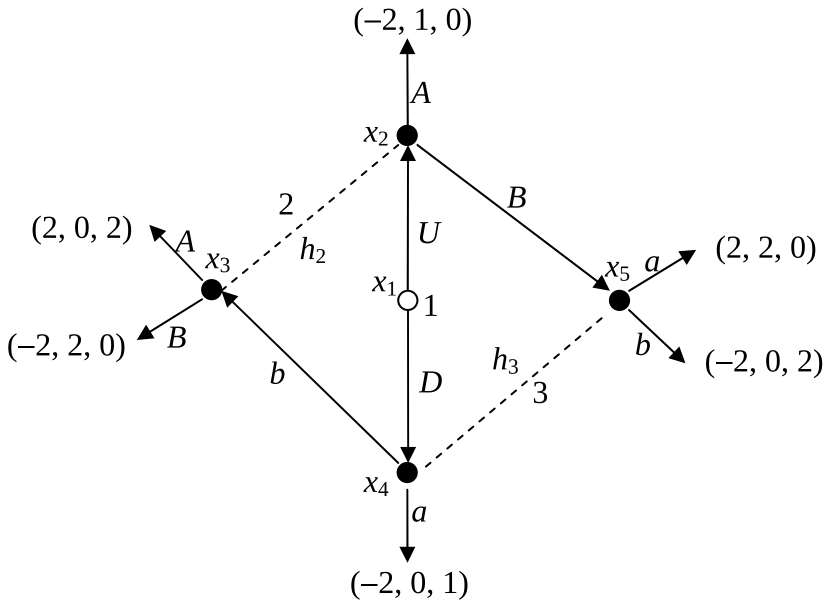
Microeconomics Comprehensive Exam

June 2009

Instructions:
(1) Please answer each of the four questions on separate pieces of paper.
(2) When finished, please arrange your answers alphabetically
(in the order in which they appeared in the questions, i.e. 1 a., 1 b. etc.).

1. Consider the following extensive-form game, whose initial node is denoted x_1:



a. Find a Nash equilibrium in pure strategies, or show that there is none.

b. Suppose that Player 1 randomizes with probability p on action U, Player 2 randomizes with probability q on action B, and Player 3 randomizes with probability r on action b. Calculate Player 2's belief $\mu_2 (x_2 \mid h_2)$ and Player 3's belief $\mu_3 (x_4 \mid h_3)$. (HINT: Are there any cases in which those beliefs are undefined?)

c. Find a perfect Bayesian equilibrium in which Player 1 randomizes between his actions with equal probability (that is, $p = ½$).

2. Two consumers, Mr. Abscissa and Ms. Ordinate, have preferences over a consumption space that consists of four bundles in $\mathbf{R}^2$: $a = (1, 1)$, $b = (2, 3)$, $c = (3, -1)$, and $d = (4, 0)$. Mr. Abscissa prefers bundle (x, y) to bundle (x', y') if and only if $x \geq x'$. Ms. Ordinate prefers bundle (x, y) to bundle (x', y') if and only if $y \geq y'$.

Given any nonempty subset of the consumption set $\{a, b, c, d\}$, Mr. Abscissa and Ms. Ordinate choose jointly – they must consume the same bundle. Their choice rule is as follows: From any one-element subset, they choose that element. For any multiple-element subset, first Mr. Abscissa eliminates one element, and then Ms. Ordinate chooses her most-preferred remaining element. (That procedure is common knowledge between the consumers, as are their preferences.)

a. Find the consumers' chosen element from each two-, three-, and four-element subset of the consumption set.

b. Find a preference ordering that rationalizes the choice rule that you found in part a, or show that none exists.

c. Suppose that an economist observes the choice behavior that you found in part a. The economist knows the procedure that Mr. Abscissa and Ms. Ordinate use to choose, and he knows Mr. Abscissa preference ordering, but he does not know Ms. Ordinate's preferences. Which (strict) preference orderings over the consumption set for Ms. Ordinate are consistent with his observations?

3. Alice's house is worthless to her if she cannot sell it. Bob and Carol are the potential buyers. Bob and Carol know their own valuations of the house, but this information is not known to anybody else. It is common knowledge that their valuations are independent and identically distributed and can take only two values: high, $v_H = \$4$(mln.), and low, $v_L = \$3$(mln.). The probability of low valuation is $p \in (0, 1)$. All agents are risk neutral. Focus only on symmetric equilibria.

a. What is Alice's first best outcome? What is her expected payoff in the first best outcome? Can Alice achieve the first best outcome?
b. What is Alice's expected payoff if she simply posts a take-it-or-leave-it price?
c. What is Alice's expected payoff in the Vickrey auction?
d. Consider the following modification of the Vickrey auction: Bob and Carol are restricted to bid only 3 or 4, and the winner pays the *average* of the winning and losing bids. Will Bob and Carol bid their true valuations? Why or why not? What is Alice's expected payoff?
e. Which of the above mechanisms will Alice choose to sell her house?

4. Consider a two-period economy ($t = 0, 1$) under uncertainty. The uncertainty in the second period is represented by the state space $\Omega = \{\omega_1, \dots, \omega_s, \dots, \omega_N\}$ ($2 \le N < \infty$) and the probability measure $\mathbb{P}$ over Ω s.t. π_s =Prob($\omega = \omega_s$) > 0 for all $s = 1, \dots, N$ ($\sum_{s=1}^N \pi_s = 1$). There is a single consumption good at each spot at $t = 1$. There is no consumption at date $t = 0$. In addition to the consumption good, there are N financial assets available for trade. The asset structure is represented by the payoff matrix $D = [d_{js}]_{j,s=1}^N$, where $d_j = (d_{js})_{s=1}^N > 0$ is the payoff vector of asset j. Further, assume that rank$D = N$.

There are $2 \le I < \infty$ agents in the economy. Each agent i is characterized by an increasing and strictly concave utility function $u_i(x)$, where x is a random variable representing consumption at $t = 1$, and a strictly positive random endowment e_i at $t = 1$. There is no endowment at $t = 0$. Assume that u_i is continuously differentiable for all i and the sets $\mathcal{U}_i(e_i) = \{x \in \mathbb{R}_+^N | u_i(x) \ge u_i(e_i)\}$ ($i = 1, \dots, I$) are closed in $\mathbb{R}_{++}^N$. Define the economy by $\left[(u_i, e_i)_{i=1}^I, D\right]$, where u_i, e_i, D enjoy the properties described above.

a. Carefully define the equilibrium for the economy $\left[(u_i, e_i)_{i=1}^I, D\right]$.
b. Characterize the equilibrium.
c. For $\mu \in \mathbb{R}_+^I \setminus \{0\}$ and $x \in \mathbb{R}_{++}^N$, define

$$U_\mu(x) = max_{(c_1,\dots,c_I)} \sum_{i=1}^I \mu_i u_i(c_i) \quad (*)$$
$$\text{s.t.} \sum_{i=1}^I c_i \le x.$$

Let $e = \sum_{i=1}^I e_i$ be the aggregate endowment. Let $\left((c_i, \theta^i)_{i=1}^I, S\right)$ be an equilibrium which you defined in part (a) and characterized in part (b). Show that there exists a $\mu \in \mathbb{R}_+^I \setminus \{0\}$ s.t. $((e, 0), S)$ is a (no-trade) equilibrium in a single agent economy $[(U_\mu, e), D]$. Moreover, show that the equilibrium consumption allocation solves (*) at the aggregate endowment, i.e., $U_\mu(e) = \sum_{i=1}^I \mu_i u_i(c_i)$.