Microeconomics Comprehensive Exam

June 2005

Instructions:

(1) The 4 problems on the exam are equally weighted. Use this information to help you maximize your score.

(2) Please answer each question on separate pieces of paper.

(3) When finished, please arrange you answers alphabetically.

1. Doctor Wisenchenko has invented a new kind of machine that will produce an uncertain income. Call this machine a "risky asset." Dr. Wisenchenko is considering selling this asset to one of two very wealthy investors by holding a first-price, sealed-bid auction with no reserve price. Wisenchenko's expected utility preferences can be represented by the function $u(y) = 2\sqrt{y}$, both investors' expected utility preferences can be represented by the utility function $v(y) = y$. There is no discounting, and the asset is indivisible.

   This problem concerns two very different strategic situations. In the first situation, which is relevant only to the first two problems, at time $t = 1$ Dr. W decides whether or not to hold the auction, which, if held, will be held at $t = 2$. At $t = 3$ the asset's return is realized and paid to whoever owns it then. These facts are common knowledge.

   a. Describe the *ex ante* efficient allocations.

   b. Describe the subgame perfect equilibria of the game.

   In the second strategic situtation, relevant only for the last two problems, the value of the parameter z is privately revealed to Dr. W, but not to the investors, at $t = 0$. The value of z is either $z_h = 20$ (with probability α) or $z_l = 8$ (with probability $1 - \alpha$). This is common knowledge.

   c. Describe all pooling perfect Bayesian equilibria. Are further assumptions need for these equilibria to exist? If so, what are they?

   d. Describe all separating perfect Bayesian equilibria. Are further assumptions need for these equilibria to exist? If so, what are they?

2. Consider an industry where a single output, cheese, is produced by households using a single input, milk. There are many households. Each household has the same maximum output capacity of cheese. Let us normalize this maximum household output to 1 and assume that it is infinitesimally small relative to the size of the cheese market. For each household, the marginal product of milk in producing cheese within that household is constant. However, that productivity differs across households. More formally, for a particular household, let b denote the amount of cheese produced per unit of milk, let z denote the milk used by the household, and let y be the amount of cheese produced. Then the household's milk production is given by

$$y = f_b(z) = \begin{cases} bz & \text{if } z \in [0, 1/b], \\ 1 & \text{if} z > 1/b. \end{cases}$$

Let w be the price of milk and p be the price of cheese.

Recall that there are many households. Assume that households' technology parameters, b, are distributed along $[1, \infty)$ according to the density function $h(b) = b^{-2}$. Hence, for any interval $[c, d]$, $1 \leq c$, the mass of households that have a parameter b lying in that interval is $1/c - 1/d$.

a. Write down the profit maximization problem for a household with productivity b. Solve for this household's milk-input demand and cheese-output supply correspondences $z_b(w, p)$ and $y_b(w, p)$. Derive its profit function $\pi_b(w, p)$.

b. Show that the total cheese industry profit is given by $\Pi(w, p) = p^2/(2w)$ if $w > p$ and $\Pi(w, p) = p - w/2$ otherwise.

c. Derive the industry's supply and demand functions for cheese and milk. (HINT: You can use the previous part, although it is not necessary.)

d. Derive the aggregate production function of the cheese industry.

3. Consider a two-period ($t = 0, 1$) pure exchange economy under uncertainty. Assume that there are $1 < \Omega < \infty$ possible states (indexed by ω) of the world in the second period , and $1 < L < \infty$ commodities (indexed by ℓ) available for consumption in each period. There are $1 < I < \infty$ agents in the economy, indexed by i. The agents have smooth, increasing, and strictly concave utility functions $u_i(x_i)$, where $x_i = (x_{li}(0), x_{li}(1), \ldots, x_{li}(\omega), \ldots, x_{li}(\Omega))_{l=1}^{L}$ is agent i's consumption. Each agent i is endowed with $e_i = (e_{li}(0), e_{li}(1), \ldots, e_{li}(\omega), \ldots, e_{li}(\Omega))_{l=1}^{L} \gg 0$ units of the consumption goods. In addition, assume that for any i, the set $U_i(e_i) = \{x \in \mathbb{R}_{++}^{L(\Omega+1)} | \, u_i(x_i) \geq u_i(e_i)\}$ is closed in $\mathbb{R}_{++}^{L(\Omega+1)}$.

   There are $J = \Omega$ financial assets (indexed by j) in zero net supply that the agents can trade at date 0. An asset j is a contract that pays $d^{j\omega} \geq 0$ in units of account in state ω, in period 1. Assume that rank $D = J = \Omega$, where D is the matrix of assets' payoffs.

   a. Carefully define financial and Arrow-Debreu equilibria for this economy.

   b. Show that if (x, θ, p, S) is a financial equilibrium, then there exists a vector $p' \in \mathbb{R}_{++}^{L(\Omega+1)}$ such that (x, p') is an Arrow-Debreu equilibrium.

   c. Show that if (x, p') is an Arrow-Debreu equilibrium, then there exist $\theta \in R^{IJ}$, $p \in \mathbb{R}_{++}^{L(\Omega+1)}$, and $S \in \mathbb{R}_{+}^{J}$ such that (x, θ, p, S) is a financial equilibrium for the economy with the asset structure defined by D.

4. There is a single seller who has a single object to sell (the seller's reservation utility is 0). There are two potential buyers, and they each value the object at 1. If the seller and buyer i ($i = 1, 2$) agree to a trade at price p in period $1 \leq t \leq 4$, then the seller receives a payoff whose present value (at $t = 1$) is $\delta^{t-1}p$, the present value of buyer i's payoff is $\delta^{t-1}(1-p)$, and buyer $j \neq i$ gets zero.

a. Consider an alternating offer bargaining with the seller choosing a buyer to make an offer to (name a price to). If the buyer accepts, the game is over. If the buyer rejects, then the game proceeds to the next period, when the buyer who received the offer in the preceding period makes a counter-offer. If the offer is accepted, it is implemented. If the offer is rejected, then the game moves to the next period when the seller makes a new proposal either to the same buyer or to the other buyer. If the offer is accepted, the game ends. Otherwise it proceeds to the last period when the buyer who received the offer in the preceding period makes a counter-offer, and the seller decides whether to accept this offer or not. If the parties do not come to the agreement, the seller keeps the object.

   Describe the subgame perfect equilibria in pure strategies and the equilibrium outcomes. Is this game different from a typical alternating offer bargaining game? Why or why not?

b. Now consider the following alternative. Suppose that if the seller rejects an offer from a buyer, he can either wait a period and make a counter-offer to this buyer, or he can *immediately* make an offer to the other buyer. Describe the subgame perfect equilibria in pure strategies and the equilibrium outcomes.