

Russell's Reasons for Logicism

Throughout his career Russell defends the strong logicist thesis that *all* of mathematics is a development of logic.¹ Nowadays, the technical details of Russell and Whitehead's strategy for attempting to demonstrate this result are well known, as are the problems upon which their project foundered. However, our improved understanding of the mathematical enterprise has yet to be matched by an adequate account of what Russell saw as the philosophical significance of his logicism. Indeed, when it comes to issues of philosophical motivation some very basic questions remain subject to controversy. Is Russell, as Nicholas Griffin has suggested, attempting to use logicism to secure the status of mathematics as a body of necessary truths, or perhaps as a body of truths knowable with certainty?² Or is he, rather, taking these features as given and attempting to explain what mathematics is, or could be, that it should have them? Is Russell, as Peter Hylton has suggested,³ using logicism in an argument against certain "idealist" views on the nature of truth? Or is he, rather, engaged in the Kantian project of investigating the ultimate grounds or "sources" of mathematical knowledge? What role might purely mathematical or methodological considerations play in motivating Russell's project? And, how does Russell's view of the philosophical significance of his logicism change as its technical complexities unfold?

The present essay is devoted to addressing these questions of philosophical motivation. Since I am chiefly interested in discovering what *first* led Russell to embrace logicism, I shall focus on his early logicist thesis, as it is presented in the 1903 *Principles of Mathematics*, but I will also consider how Russell's reasons for advancing logicism evolve in the years leading up to *Principia Mathematica*. I shall argue that Russell's chief purpose, initially at least, is to provide an account of the fundamental *nature* of mathematics; or, more precisely, of *what* is known when we have mathematical knowledge. This question concerns both the epistemological ground of mathematical knowledge and the subject-matter of mathematical propositions. As we shall see (in section 3), for Russell these questions can come apart. I shall argue that Russell views logicism as the only theory of the nature of mathematics that is compatible with what he takes to be its uncontroversial status as a body of certain and exact knowledge. Russell's project is thus more explanatory than polemical, and it is not directed exclusively against idealism. Although Russell's conception of his project evolves over time, what changes is not its general explanatory orientation. After 1906, Russell no longer conceives

of the logicist reduction as explaining the certainty of mathematical knowledge, but he does see it as having the advantage over its competitors of accounting for its *exactitude* and *truth*.

My hope is that a consideration of these issues will help to illuminate a number of central aspects of Russell's philosophy of mathematics, including his views on: theoretical unification, rigorization, self-evidence, and proof. A further goal will be to clarify the role played by logicism in Russell's opposition to Kant, Hegel, and the British neo-Hegelians. I shall argue that although Russell was later to report that he had first thought of logicism as "a *parenthesis* in the refutation of Kant,"⁴ in fact logicism *per se* plays a less important role in Russell's opposition to Kant than is often supposed.⁵ At least as significant in this regard, I shall argue, is his "if-thenist" *analysis* of mathematical propositions. Moreover, Russell's opposition to *post-Kantian* idealism makes no appeal to logicism at all. What motivates Russell's rebellion against the monistic idealism of Bradley, Joachim and others, I shall argue, is a set of purely metaphysical arguments. But before we begin it will be useful to remind ourselves of the main features of Russell's early logicism.

1. Russell's early logicism

Although Russell occasionally formulates his early logicism as the thesis that mathematics *is* symbolic logic (*Principles*, § 4), the claim he really wishes to defend is the weaker thesis that mathematics is a "development" of logic (*Principles*, § 120). "All mathematics," he says, "is deducible...from the primitive propositions of formal logic" (*Principles*, § 434) and "there are no primitive propositions in mathematics except those of logic" (*Principles*, § 412). The task Russell sets himself has two parts. First, it must be shown that the propositions we pre-theoretically regard as propositions of "pure" or "non-applied" mathematics are propositions of "pure mathematics" in Russell's proprietary sense, that is to say: generalized conditionals containing none but logical vocabulary in which a bound variable has occurrences on both sides of the conditional (cf. *Principles*, § 1). Second, it must be shown that these conditionals follow from logic alone. The first step involves an analysis of mathematical propositions: one defines the relevant non-logical vocabulary in purely logical terms, and one paraphrases the sentences in question in such a way as to bring out the conditional form of the propositions they express. So, for example, the numerical formula " $1 + 1 = 2$ " receives a preliminary analysis along (roughly) the following lines: "If the number belonging to x is 1 and the number

belonging to y is 1 and x and y are disjoint, then the number belonging to the union of x and y is 2.”⁶
The numerals occurring here are then defined in set-theoretic terms.

Russell would later come to regard his treatment of the generalized conditional as *the* form of a proposition of pure mathematics as misjudged.⁷ What had led him to this view, he would later explain, was the need to acknowledge both Euclidean and non-Euclidean geometry as legitimate branches of mathematics.⁸ Because only one of these systems could be true of actual space, the theorems of pure geometry would have to be conceived of as making no claims about its nature. The solution was to construe those theorems as conditionals of the form: “if such and such geometrical axioms hold of a given space, then so and so properties characterize that space” (cf. *Principles*, § 5). Such a reconceptualization of geometry leads to a bifurcation of the traditional subject:

Geometry may be considered [either] as a pure *a priori* science, or as the study of actual space. In the latter sense, [it is] an experimental science, to be conducted by means of careful measurements....As a branch of pure mathematics, Geometry is strictly deductive, indifferent to the choice of its premises and to the question whether there exist (in the strict sense) such entities as its premises define. (*Principles*, § 352).

The upshot is that Russell’s logicism about geometry can profitably be regarded as comprising two separate components:

i) A proof that conditionals of the form “ $\forall x (\text{Geom Ax.}(x) \supset P(x))$ ” are derivable from the axioms of logic (cf. *Principles*, § 5).

(Where “Geom Ax.(x)” is a predicate asserting of an entity that it is characterized by the axioms of a particular system of geometry, and “ $P(x)$ ” is a suitably parametrized counterpart of a theorem of that system.)

ii) Definitions in purely logical terms of all the (apparently) non-logical vocabulary involved in these conditionals.⁹

These two components are mutually independent, but Russell’s logicism about geometry involves them both. For this reason, the suggestion sometimes made that Russell is as much a logicist about “pure economics” or “pure geography” as he is about pure geometry must be rejected.¹⁰ For, although Russell might have entertained an if-thenist conception of these non-mathematical disciplines, he made no attempt to define their vocabulary in purely logical terms.

Considerations of symmetry might lead one to expect Russell’s logicism about *arithmetic* to take the form of a thesis about the derivability from pure logic alone of conditionals of the form: “ $\forall x (\text{Peano Ax.}(x) \supset P(x))$,” together with a thesis about the definability in logical terms of their apparently non-logical vocabulary (Here “Peano Ax.(x)” is a predicate saying that an entity realizes the structure specified by the Peano’s axioms; and “ $P(x)$ ” is a suitably parametrized counterpart of a theorem of arithmetic¹¹). But, in practice, Russell fails to sustain the parallel. With respect to arithmetic, he entertains the more ambitious goal of deriving the Peano axioms themselves from the primitive propositions of symbolic logic together with suitable definitions (*Principles*, § 120). Why does Russell treat arithmetic and geometry so differently? The answer, I believe, is that Russell regards the proof of Peano’s axioms as mathematically feasible, and he sees no reason why they should not be regarded as *the* axioms of arithmetic. Arithmetic, after all—unlike geometry—had not given rise to mutually incompatible yet internally consistent systems.

This will suffice as a preliminary exposition of the elements of Russell’s early logicism, but before we turn to questions of motivation it will be convenient to establish some shorthand. By “if-thenism” I shall mean a view about the *content* of mathematical claims: namely, that they take the form of generalized conditionals. By “rigorism” I shall mean the view that the consequents of these conditionals are derivable from their antecedents by rigidly deductive reasoning. (Because Russell tacitly assumes the deduction theorem, rigorism for him is equivalent to the view that these conditionals are themselves provable from logic alone.) By “definitional logicism” I shall mean the view that all notions of what is commonly called “pure mathematics” are definable in logical *cum* set-theoretic terms. Finally, by “logicism” *sans phrase*, I shall mean the result of combining definitional logicism with rigorism. These definitions will, of course, need to be understood as suitably modified when logicism merely about a single branch of mathematics is in question.¹²

2. Motivations I: The critique of Kant

The aim of the present section is to identify the anti-Kantian implications of Russell’s logicism. I shall argue that although logicism does play a role in Russell’s critique of Kantianism, what Russell identifies as the most important elements of that critique make no essential appeal to it.

Russell takes Peano’s “new science” of Symbolic Logic to make possible the “final and irrevocable” refutation of the Kantian view of mathematics (*Principles*, § 4). He sees the cardinal

feature of Kant's view as being its willingness to treat deductively invalid modes of reasoning as *mathematically* valid: "What is essential [in Kant's view of mathematics], from the logical point of view, is that the *a priori* intuitions [of space and time] supply methods of reasoning and inference which formal logic does not admit; and these methods, we are told, make the figure (which may of course be merely imagined) essential to all geometrical proofs." (*Principles*, § 433). So, for example, Russell takes Kant's appeal to intuition in geometry to be motivated by the need to plug the gaps in Euclid's logically invalid proofs:¹³ "The actual propositions of Euclid...do not follow from the principles of logic alone; and the perception of this fact led Kant to his innovations in the theory of knowledge." (*Principles*, § 5). Since Peano's symbolic logic provides the means for giving gap-free proofs, and since in Russell's view such proofs can be given, its adoption serves—from Russell's point of view—to eliminate one motivation for invoking Kantian intuition. Russell, however, is well aware that Kantian intuition might be thought to play the rather different role of grounding the *axioms* of geometry: "[Even] admitting the *reasonings* of Geometry to be purely formal, a Kantian may still maintain that an *a priori* intuition assures him that the definition of three-dimensional Euclidean space, alone among the definitions of possible spaces, is the definition of an existent..."(*Principles*, § 434). So, even if Kant's view of geometrical reasoning (as essentially diagrammatic) should receive a conclusive refutation, Kant might still insist on the need for an appeal to *a priori* intuition in verifying geometrical axioms. Russell himself doubts that *a priori* intuition plays such a role (*Principles*, § 434), but the crucial point for him is that his "if-thenism" in any case, renders *this* question moot: "[The view that an *a priori* intuition assures us that actual space is three dimensional and Euclidean] is, strictly speaking, irrelevant to the philosophy of mathematics, since mathematics is throughout¹⁴ indifferent to the question whether its entities exist." (*Principles*, § 434; cf. § 352).

So, the nub of Russell's disagreement with Kant on geometry is not a dispute over the true grounds of our knowledge of the geometry of actual space, but rather a dispute over the true grounds of our knowledge of conditionals of the form " $\forall x (\text{Geom Ax.}(x) \supset P(x))$." For Russell, the crucial question is whether these conditionals can be derived from logic alone. But, from Russell's perspective—since he tacitly assumes the deduction theorem¹⁵—this reduces to the question whether geometrical *reasoning* is purely logical. Consequently, in order to refute Kant's views on the nature of geometrical reasoning it suffices—from Russell's point of view—merely to formulate Euclidean geometry as a rigorous deductive theory. There is no need to define "point," "line," "plane," etc., in logical terms, and so no need for definitional logicism. (Hence no need for logicism *sans phrase*)

Summing up his discussion of Kant, Russell says: “It is now proved (what is fatal to the Kantian philosophy) that every Geometry is rigidly deductive, and does not employ any forms of reasoning but such as apply to *Arithmetic and all other deductive sciences*.” (*Principles*, § 353, emphasis added). This remark suggests that Russell simply *takes it for granted* that the *reasonings* of *arithmetic*—as opposed to its axiomatic basis¹⁶—are uncontroversially logical in character. Since he does not discuss Kant’s views on algebra, we may conclude that the *only* point at which Russell sees the historical Kant as posing a challenge to the deductive character of mathematical reasoning is in geometry. If that is right, it follows that Russell’s logicism (as opposed to his rigorism) plays no essential role in his rebuttal of (what he takes to be) Kant’s view of mathematical *reasoning*.

Where logicism is indispensable to the refutation of Kantianism—if not of Kant himself—is in showing that the axioms of *arithmetic* are not grounded in temporal intuition (*Principles*, § 433, cf. § 434). However, it is debatable whether Russell really has a disagreement with the historical Kant on this point, since Kant denies that arithmetic has any axioms,¹⁷ and, arguably, he sees time (or our intuition of it) as grounding not arithmetic but *pure mechanics*.¹⁸ That said, Russell does have a genuine disagreement with Kant about this last science. As he notes, Kant’s view that the calculus has an essential reference to time (or, at least to space or time), draws its strength from the belief that the notion of continuity is explicable only by reference to continuous change in space and time (*Principles*, § 249). He observes that the new mathematics, with its explicit definition of continuity, makes such an appeal unnecessary. We may conclude that for Russell the arithmetization of analysis—but not logicism—plays a role in combating Kant’s view of the epistemological ground of pure mechanics.

It now becomes clearer why Russell should have once thought of logicism as a mere “parenthesis” in the refutation of Kant.¹⁹ The parenthesis, I take it, is the use of logicism about arithmetic to refute Kant’s view—or what Russell takes to be Kant’s view—that this science in some way depends on time. In Russell’s opinion, what is more central to the refutation of Kant is the demolition of what he calls “two pillars of the Kantian edifice” (*Principles*, § 433), namely, the doctrine that the *reasonings* of mathematics are different from those of formal logic, and the idea that there are contradictions in the ideas of space and time (*ibid.*). We have just seen that Russell’s rebuttal of the first of these views makes no essential appeal to logicism. In section 5 I shall argue that neither does his rebuttal of the second.

One loose thread still remains: we have yet to explain why Russell should have been a logicist, rather than merely a “rigorist,” about geometry. If anti-Kantianism does not motivate this aspect of Russell’s view, what does? The answer, I believe, is that Russell is concerned to show that geometry really is a branch of “*pure* mathematics” in his technical sense. If Euclidean geometry, for example, were to be a body of conditionals containing non-logical vocabulary, then, although its deductive *basis* might indeed be logical, it would remain, in virtue of containing non-logical constants, merely a branch of *applied* mathematics (cf. *Principles*, § 9). Since Russell’s aim is to establish *what* geometry is, and not just how it is epistemically grounded, and since it is to be shown to be, at root, logic, its basic concepts must be shown to be definable in purely logical terms.

3. Motivations for Logicism II: the nature of mathematics

Having considered the limited anti-Kantian significance of Russell’s logicism, it is time to turn our attention to motivations for Russell’s logicism that go beyond opposition to Kant. Significant here is a piece of stage-setting from the opening of the *Principles*:

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[Hitherto while] it was generally agreed that mathematics is in some sense true, philosophers disputed as to what mathematical propositions really meant: although something was true, no two people were agreed as to what it was that was true, and *if something was known, no one knew what it was that was known*. So long, however, as this was doubtful, *it could hardly be said that any certain and exact knowledge was to be obtained in mathematics*. (*Principles*, § 3, emphases added)

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On first acquaintance, is easy to hear this passage as making the implausible claim that until a correct theory of the nature of mathematical knowledge is arrived at, such knowledge can be neither certain nor exact. But the continuation of the passage suggests a more palatable idea:

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We find, accordingly, that idealists have tended more and more to regard all mathematics as dealing with mere appearance, while empiricists have held everything mathematical to be approximation to some exact truth about which they had nothing to tell us. This state of things, it must be confessed, was thoroughly unsatisfactory. (*Principles*, § 3)

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Russell's real point in *Principles*, § 3 is that the going theories of the nature of mathematics are incompatible with the certainty and "exactness" (i.e., non-approximate character) of mathematical knowledge, with the consequence that, so long as these theories remain unrefuted our common sense opinion that we *do* have certain and exact mathematical knowledge will seem unjustified. This reading is supported by a remark Russell makes in his 1911 lecture "Analytic Realism":

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From time immemorial it has been agreed that mathematics is as certain as any body of knowledge can be and yet every philosopher has come to the conclusion that mathematics is either completely erroneous or tainted by the inaccuracies of sensory knowledge. (*Papers*, v. 6, 137)

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This claim is unpacked by an earlier remark from the same lecture:

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Idealism, taken seriously, leads to the conclusion that what is called "*a priori* truth" is nothing but an illusion; one cannot help but believe it, but it is not what is the case. On the other hand, empiricism, by which we mean the theory that all evidence rests on sense-experience, leads to the conclusion that one cannot know anything except sense-data....Therefore, neither idealism nor empiricism provides a theory of knowledge consistent with the facts (*Ibid.*, 136–7).

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What facts? The remark just quoted from *Principia*, § 3 suggests that the facts in question are the facts, first, that mathematics is true, and, second, that it affords a body of exact and certain knowledge. Russell is claiming that empiricism and idealism are incompatible with these evident features of mathematics. His idea is that empiricism poses a threat to the exactness of mathematical knowledge because it leaves mathematics looking as though it holds only within the limits of empirical observation. The empiricist must maintain, for example, that if our best empirical science cannot decide between Euclidean and non-Euclidean geometry as the truth about actual space, the most we can know is that the internal angles of any observed triangle sum to *approximately* two right angles. Empiricism, therefore, threatens the exactness of mathematical knowledge not in the first instance because it can only provide vague definitions of mathematical concepts—though that may be so, but because wherever observations fail in principle to decide between two competing hypotheses, the mathematical facts must be treated as *irreducibly approximate*.

Idealism, on the other hand, is taken to threaten the *certainty* of mathematics. One way in which it does so, according to Russell, is that it threatens the status of mathematical claims as eternal truths: “It might happen, if Kant is right, that tomorrow our nature would so change as to make two and two become five. This possibility seems never to have occurred to [Kant], yet it is one which utterly destroys the certainty and universality which he is anxious to vindicate for arithmetical propositions.”²⁰ In the *Principles* Russell emphasizes the threat Kantianism poses to the *truth* of mathematics:

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Concerning necessities of thought, the Kantian theory seems to lead to the curious result that whatever we cannot help believing must be false. What we cannot help believing, in this case, is something as to the nature of space, not as to the nature of our minds. The explanation offered is, that there is no space outside our minds; whence it is to be inferred that our unavoidable beliefs about space are all mistaken. (*Principles*, § 430)

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We might reconstruct Russell’s reasoning as follows: Kant’s ground for regarding our knowledge of space as necessary—and so as *a priori*—is, at bottom, the unavoidability of our beliefs about space. The reason why these beliefs are unavoidable, for Russell’s Kant, is that space is in fact subjective. Yet it is part of the content of these beliefs that they represent space as objective. So, according to Russell, Kant’s very explanation of the *a prioricity* of geometry commits him to the falsehood of these same supposedly *a priori* beliefs.

In his 1902 article “The Study of Mathematics”²¹ Russell adds a version of psychologism to the list of positions he takes to misrepresent the nature of mathematics:

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Philosophers have commonly held that the laws of logic, which underlie mathematics, are laws of thought, laws regulating the operations of our minds. By this opinion the true dignity of reason is very greatly lowered: it ceases to be an investigation into the very heart and immutable essence of all things actual and possible, becoming, instead, an inquiry into something more or less human and subject to our limitations. (*Mysticism and Logic* (Totowa, New Jersey: Barnes and Noble, 1981), 55)

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Taken together, these passages suggest that Russell's aim in the *Principles* is to provide the first adequate account of the nature of mathematical knowledge, or more precisely, the first adequate account of *what* is known when we have mathematical knowledge—an account that must explain the special character we take mathematical knowledge to have.

For Russell, achieving clarity about the *content* of what is known is just as important as discovering its epistemic ground. Russell wants to make clear the *meaning* of mathematical statements (*Principles*, § 3). He wants to explain what mathematics really *is*.²² This task calls for an analysis of the concepts with which mathematics deals. It is important to realize that, owing to Russell's if-thenism, these two questions—the “meaning” or subject matter of a science, on the one hand, and its epistemic ground, on the other—can come apart. A proposition of “pure physics,” “pure biology,” or “pure economics,” in the sense discussed in section 1, will deal with non-logical subject matter, but, so long as this science can be rigorized, its epistemic ground will be logic.

So to sum up, logicism is presented in the *Principles* as the best available account of the character of our mathematical knowledge. If logicism could be established, we would have one explanation of how mathematics could be the body of true, certain, and exact knowledge we take it to be (always assuming, as Russell does, that *logic* has these features). Russell supposes that all rival theories of the nature of mathematics are incompatible with its having this character. His intentions, then, are not exclusively anti-Kantian or anti-idealist, for empiricism and psychologism are as much his targets as Kantianism and idealism.

4. Motivations for logicism III: Axiomatics

Russell's interest in discovering the fundamental basis of mathematics is independent of, and historically prior to, his conviction that its basis is logical. In his letter to Couturat of 18 July 1898, Russell describes his task as “the discovery of the fundamental ideas of mathematics and the necessary judgements (axioms) which one must accept on the basis of these ideas.”²³ But he says nothing at this time to suggest that he expects the fundamental ideas and principles of mathematics to be logical in character. On the contrary, before becoming familiar with Peano's Symbolic Logic in 1900, Russell had flatly rejected such an idea:

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Until I got hold of Peano, it had never struck me that Symbolic Logic would be any use for the Principles of mathematics, because I knew the Boolean stuff and found it useless. It was Peano's ϵ , together with the discovery that relations could be fitted into this system, that led me to adopt symbolic logic.²⁴

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Logicism, then, was the position Russell arrived at in attempting to answer a question he had already posed, namely: what are the fundamental ideas and principles of mathematics? That being so, part of understanding Russell's motivations for his logicism will involve appreciating why he thought axiomatics worth pursuing on its own account. The answer to this question is multi-layered.

First, and most straightforwardly, Russell regards theoretical unification as possessing a kind of intrinsic aesthetic value: "The discovery that all mathematics follows inevitably from a small collection of fundamental laws is one which immeasurably enhances the intellectual beauty of the whole."²⁵ Second, Russell believes that the goal of seeking the smallest number of basic principles from which mathematics may be derived has instrumental value. Because new areas of mathematics may be opened up when an attempt at unification *fails*, the *demand* for unification can be mathematically fruitful. The case of non-Euclidean geometry—or "Metageometry" as Russell terms it²⁶—serves as an illustration. Russell observes that: "It was by the refusal to admit [the axiom of parallels] without proof that Metageometry began."²⁷ The point is amplified in the *Principles*:

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Having found that the denial of Euclid's axiom of parallels led to a different system, which was self-consistent and possibly true of the actual world, mathematicians became interested in the development of the consequences flowing from other sets of axioms more or less resembling Euclid's. Hence arose a large number of Geometries, inconsistent, as a rule, with each other, but each internally self-consistent.²⁸

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In other words, in the case of geometry, failure to prove a dependence result led to the discovery of a new consistent set of axioms, which, in turn, led to the investigation of further systems of non-Euclidean geometry.

Third, Russell regards the demand for proof of truths formerly considered self-evident as sometimes possessing a kind of diagnostic value. He holds that a persistent failure to prove something *when the means for proving similar results has been developed* may, in certain cases, afford defeasible

evidence of the falsehood of what one is trying to prove.²⁹ Consequently, “One of the merits of a proof is that it instils a certain doubt as to the result proved; and when what is obvious can be proved in some cases, but not in others, it becomes possible to suppose that in these other cases it is false.”³⁰ The *Principles* contains a concrete illustration of this point: “As soon as it was found that the similarity of whole and part could be proved to be impossible for every finite whole, it became not implausible to suppose that for infinite wholes, where the impossibility could not be proved, there was in fact no such impossibility” (*Principles*, § 341). By “similarity of whole and part” Russell means the one-one correlation of the members of a set with the members of one of its proper subsets. His point is that by framing precise set-theoretic definitions of “similarity,” “whole,” and “part,” mathematicians acquired, for the very first time, the means to *prove* for the finite case the principle that no whole is similar to its (proper) part. Since an analogous result could not be proved for the infinite case, it became possible to suppose that in that case the result might not even be true. In this way, the demand for proof, when combined with the existence of definitions that (in some cases) made proof feasible, created a reason to be sceptical about a principle that had previously proved impervious to doubt. Since the principle in question was in fact false, this represented a mathematical advance.³¹

These considerations suggest that Russell’s interest in logicism has motivations that are independent of his desire to refute Kant. But it is worth noting, further, that even Russell’s interest in *rigor* extends beyond his anti-Kantianism. In Russell’s opinion, rigorization is of value not only to the philosopher interested in the grounds of a subject, but also to the working mathematician. For Russell, a “pedantic” insistence on making explicit all the principles that licence the steps in our deductions is of value because it can bring to light unconsciously made, yet mathematically important, assumptions.³² Unearthing these assumptions is of value in itself, but it can also serve to advance mathematics into new regions: “By banishing the figure [and using reasoning that proceeds by strict rules of formal logic from a set of axioms laid down to begin with], it becomes possible to discover all the axioms that are needed; *and in this way all sorts of possibilities, which would have otherwise remained undetected, are brought to light.*”³³(emphasis added). Russell means, I take it, that the explicit formulation of previously overlooked assumptions can bring to light new possibilities because by making these assumptions explicit one makes it possible to consider cases in which they *fail*. An illustrative example would be the proof of Euclid’s first proposition, in which one constructs an equilateral triangle on a given base. In the course of this proof, one tacitly assumes that the circles drawn have a point of intersection. When this assumption is added as an explicit axiom, it becomes

apparent that it fails in an “elliptical” space, such as the surface of a sphere, when the base of the triangle to be constructed is sufficiently long.³⁴

This concludes our survey of the reasons behind Russell’s logicism. I have emphasized the importance of seeing Russell’s anti-Kantianism as only one of a range of motivations. But it remains to be asked whether logicism might have played an important role for Russell in a broader anti-idealist argument. It is to this question that I now turn.

5. Logicism and Idealism

In his landmark study, *Russell, Idealism, and the Emergence of Analytic Philosophy*, and elsewhere,³⁵ Peter Hylton has argued that Russell has a narrowly metaphysical motive for defending logicism. Hylton maintains that “For Russell in the early years of [the twentieth century]...logicism was the basis for a complex argument against idealism, of both the Kantian and the non-Kantian varieties.”³⁶ It is central to Hylton’s interpretation to regard Russell’s logicism as motivated *exclusively* by these anti-idealist goals. Accordingly, he denies any role for epistemological considerations in motivating Russell’s project: “Russell,” he says “completely rejects the Kantian concerns with the sources of knowledge, and with anything recognizable as epistemology at all.”³⁷ Hylton finds in Russell two separate anti-idealist arguments that use logicism as their basis. First, and most explicitly, “Logicism shows that consistent theories of space and time are available; the spatio-temporal world need not be written off as contradictory or not fully real”.³⁸ Second, there is a less explicit argument to the effect that “mathematics functions as a particularly clear counterexample to the crucial idealist claim”³⁹ that “all our ordinary knowledge (of science, history, mathematics, etc.) is true, at best, in a conditioned and non-absolute sense of truth.”⁴⁰

In the present section I will examine this interpretation in some detail. I do so for two reasons: First, insofar as Hylton rejects any role for epistemology in motivating Russell’s logicism, his reading poses a serious challenge to my own interpretation. Second, Hylton’s reading presents the most plausible case I know that Russell’s logicism has anti-(neo) Hegelian as well as anti-Kantian motivations.

It will be useful to begin by making some observations about what Hylton takes to be Russell’s “more implicit” argument. First, Hylton understands the “crucial idealist claim” to be restricted to what he calls “ordinary knowledge.”⁴¹ The restriction is necessary because Hegelian idealists allow

that some knowledge *is* true in an unqualified sense—namely: so called “metaphysical knowledge” of “the world as a whole.” Second, there are several ways in which the truth of a known proposition might be considered “conditioned” or “non-absolute.” Our ordinary knowledge might be held to be true only “relative to a particular viewpoint,” or only “relative to a certain stage in a dialectic” or, finally, “[only] if put in a wider context.”⁴² It is in order to secure the “unconditioned” or “absolute” truth of mathematics against this whole family of idealist views that Russell develops his logicism—or so Hylton contends. The last point to note is that Hylton’s Russell does not use logicism merely defensively, as a way of satisfying himself that mathematics is absolutely true, but rather polemically: Russell’s logicist critique of Kant “[serves] as the basis for a *general attack* on Kantianism and post-Kantian idealism.”⁴³

Hylton does not explain precisely how logicism figures in this broadly anti-idealist argument, but he makes a number of remarks that suggest the following reasoning:⁴⁴ It is accepted on all sides that *logic*, at least, is unconditionally and absolutely true. Logicism then extends the unconditional and absolute truth of logic to one branch of “ordinary” human knowledge—namely, to pure mathematics. This generates a counterexample to the “crucial idealist claim” about truth.

This argument seems to me to face two powerful objections. First, there is simply no indication in the text that this is Russell’s intended strategy. Secondly, it is doubtful that Russell could have intended to argue this way. The problem is as follows: Because the argument is intended to have *suasive* force against idealists, they too must be prepared to grant its assumption that logic is unconditionally true.⁴⁵ But, although *Kant* might have accepted this assumption, it unclear that post-Kantian idealists in general would have done so.

Consider, for example, F. H. Bradley. In his *Appearance and Reality* Bradley argues that the abstract truths of mathematics, being “[relatively] remote from fact, more empty and incapable of self-existence” are therefore “less true.”⁴⁶ And he asserts, more generally, that “the law which is more abstract contradicts itself more” (ibid). Since the laws of logic are plausibly among the most abstract truths there are, it is at least questionable whether Bradley would have accorded them the highest degree of truth. Having said this, Bradley also maintains that “the more concrete connections of life or mind” since they “hold good over a less[er] extent of reality” are “more false” (ibid.). So his two criteria for being “more true”/ “less false”—namely, being (a) more concrete and (b) more extensive in application—point in opposite directions. The picture is blurred, but we can gain some clarification by

examining what Bradley says about one particular law of logic. Concerning the Law of Contradiction, he remarks:

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Against my intellectual world the Law of Contradiction has ... claims nowhere satisfied in full. And since, on the other hand, the intellect insists that these demands must be and are met, I am led to hold that they are met in and by a whole beyond the mere intellect. And in the intellect itself I seem to find an inner want and defect and a demand thus to pass itself beyond itself. (Bradley, *Appearance and Reality*, 508)

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Since the Law of Contradiction does not fully satisfy the claims of the intellect, it seems that it cannot be wholly true. For “truth has to satisfy the intellect, and ... what does not do this is neither true nor real” (Bradley, *Appearance and Reality*, 509).

Whether Russell would have read Bradley in the way I have done is unclear,⁴⁷ but he does appear to have appreciated that the neo-Hegelians more generally would not have regarded logical laws as “wholly true.” In his article “On the Nature of Truth” of 1907,⁴⁸ Russell discusses the neo-Hegelian ideas of Harold Joachim, a colleague of Bradley’s at Merton College, Oxford, whose 1906 book, *The Nature of Truth*,⁴⁹ Russell regarded as representative of turn-of-the-century British neo-Hegelianism.⁵⁰ Joachim maintains that *no* judgement considered in isolation is absolutely true. Instead, a judgement approaches a higher degree of truth the more comprehensive, hence coherent, the system of knowledge to which it belongs.⁵¹ Thus:

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Actual human knowledge, is never completely self-coherent, if only for the reason that it is growing in time. And if the ideal is never fully embodied in the whole of actual human knowledge, *a fortiori* it refuses to dwell entire within “a science,” and of course *minime* within a single judgement. [Joachim, *Truth*, 114]

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Presumably then, the axioms of any known logical system cannot be wholly true. For they comprise only a small part of the system they ground, and this system itself constitutes only one branch of human knowledge. Moreover, the terms in which Russell discusses Joachim’s view suggest that he would have been well aware of this point. Russell encapsulates Joachim’s position in the slogan that

“nothing is wholly true except the whole truth.”⁵² Since Russell is unlikely to have taken Joachim to believe that logic was the whole truth, it is doubtful that he would have taken him to believe that it was wholly true.

Could Russell’s argument against *Kant* have proceeded in the manner Hylton envisages? Again, this seems doubtful. For, as we have seen, Russell attacks Kant, not by using logicism to establish that mathematics is true, but by appealing to the evident truth of mathematics as a ground for rejecting any theory, including—on Russell’s interpretation—Kant’s own, according to which mathematics is not true.⁵³ In short, the truth of mathematics is a point of departure for Russell, not a destination.⁵⁴

Let us turn, then, to what Hylton considers to be Russell’s more explicit argument. This is an attempt to implicate logicism in a defence of the adequacy and coherence of the notions of space and time against the challenge posed by Kant’s “mathematical” antinomies.⁵⁵ Hylton reconstructs Russell’s argument as follows: First, drawing on the “the treatment of real numbers and of continuity made available by the work of Cantor, Dedekind, and (especially) Weierstrass,” Russell establishes that there are “consistent mathematical theories of space and time.”⁵⁶ Second, Russell invokes logicism to demonstrate that these mathematical theories are independent of space and time.⁵⁷ Hylton sees this second step as essential: “It is only if mathematics is in this way independent of space and time that it can be used, in a non-circular fashion, as an argument for the consistency of the latter notions.”⁵⁸ Again, Hylton’s presentation is compressed and it is hard to be certain how the details of the argument are supposed to unfold, but we may perhaps construct a cogent argument along the lines he suggests, at least for the case of time.⁵⁹ Plausibly, in order to establish the consistency of the concept of time, Russell would need to formulate a theory of time and show that this theory had a model. So one might have expected him to formulate an axiomatized theory of time (as, say, a Dedekind complete, dense linear ordering without endpoints) and then to find a model for this theory. If he had thought of the real numbers as providing such a model, he might then have worried that the theory of reals could itself be essentially dependent on time, and so still potentially inconsistent. Had he reasoned this way, Russell might have appealed to a logicist reduction of the theory of the reals in order, in effect, to provide a relative consistency proof for this theory.

But whatever the merits of this line of argument, I would hesitate to endorse it as exegesis. It is a good account of how Russell might have attempted to use logicism, but as an interpretation it suffers from a lack of textual support. Russell in fact never formulates a comprehensive axiomatized theory of

time (or space) in the *Principles*. And he certainly does not adduce anything as a model for such a theory. Instead, his actual defences of the consistency of space and time follow an entirely different pattern. Russell attempts to show that the reasoning appealed to in Kant's and Zeno's arguments for the inconsistency of space and time rests on false mathematical assumptions. Kant's first antinomy, for example, is said to rest on a denial of two principles validated by Cantorian set theory: first, that infinite sets exist, and, second, that sets can be specified without enumerating their members (*Principles*, § 435). Zeno's paradox of Achilles and the tortoise, for its part, is resolved by appealing to Cantor's principle⁶⁰ to show that although the tortoise visits just as many points as Achilles, his path can nevertheless be a proper part of Achilles' (*Principles*, § 331). Thus Russell defends the consistency of the spatio-temporal continuum merely by attempting the piecemeal rebuttal of arguments for its inconsistency, and he treats these rebuttals as together constituting a *sufficient* defence of the consistency of space and time (see *Principles*, § 335). There is never any suggestion that a positive, wholesale argument for consistency is either necessary or attainable.

Nor does Russell voice any suspicion that the purely mathematical continuum might itself rest on space or time. What Russell *does* concern himself with is an argument that the mathematical continuum contains a contradiction (*Principles*, § 336). However, he sees this argument as resolved, not by invoking a logicist reduction as a relative consistency proof of the theory of the reals, but simply by denying the existence of "infinitesimal segments" (*ibid*). In other words, the arithmetization of analysis, with its elimination of the notion of an infinitesimal, is invoked to show that the mathematical continuum does not involve the contradictory idea of a number that is smaller than any other number but not identical to zero (*Principles*, § 309). Having thus identified the source of the apparent contradiction in the notion of the mathematical continuum, Russell entertains no further suspicion that the theory of the reals might harbour a contradiction. I conclude that it is doubtful that Russell intends to use logicism as a means for establishing the consistency of space or time.

But before moving on, I should comment on two passages that can seem, at first glance, to provide strong support for Hylton's reading. The first is a remark Russell makes in the *Principles* in the course of summarizing the main points of his discussion of Kant:

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We saw that ... Kant's belief in the peculiarity of geometrical reasoning, and in the existence of certain antinomies peculiar to space and time, has been disproved by the modern realization of Leibniz's universal characteristic. (*Principles* § 436)

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If the phrase “the modern realization of Leibniz’s universal characteristic” could be shown to refer to the thesis of logicism, this remark would provide strong support for Hylton’s reading. However, such a reference appears unlikely, since Russell elsewhere represents Leibniz’s Universal Characteristic as the forerunner, not of logicism, but of *Peano’s formal system*. In his article “Recent Work on the Principles of Mathematics,”⁶¹ for example, Russell says that ‘Leibniz foresaw the science which Peano has perfected, and endeavoured to create it’⁶² and he terms this science Leibniz’s “Universal Characteristic.” The science that Peano had “perfected,” however, was not a logicist reduction of arithmetic, but merely a formal system purporting to be adequate for the derivation of the truths of arithmetic from the Dedekind-Peano axioms. Moreover, Russell knew this. In his 1901 essay “Recent Italian Work on the Foundations of Mathematics”⁶³ he notes that, although Peano aspired to define the individual numbers in “purely logical” terms,⁶⁴ and although he had *conjectured* that arithmetic could be developed with the aid of these definitions, in practice he treated zero, successor and (finite) number as primitive notions. Peano, he says, did not attempt a logicist reduction, but merely “content[ed] himself with indicating ... that arithmetic can be developed [when Peano’s primitives are defined in set theoretic terms].”⁶⁵ Furthermore, Russell is explicit that Peano was in no position to substantiate his logicist conjecture. Speaking of Peano’s three primitives, Russell says: “Even these three can be explained by means of the notions of relation and class; but this requires the Logic of Relations, which Professor Peano has never taken up.”⁶⁶ In the light of these considerations, I find it doubtful that the phrase “The science which Peano...*perfected*” (emphasis added) could be meant to refer to a logicist reduction of arithmetic.

But how does Peano’s system contribute to the refutation of Kant, if not by providing a logicist reduction? The answer, I believe, is that, since it is a system in which—ideally at least—claims are made unambiguously, and in which all the premises and rules on which a proof relies are explicitly recorded, Peano’s system has the power (1) to facilitate gap-free proofs in geometry, thus refuting Kant’s view of geometrical reasoning as essentially diagrammatic, and (2) to render explicit the fallacious set-theoretic assumptions on which Kant’s mathematical antinomies rest.

Of course, it remains possible that Peano’s system could be said to refute Kant’s views because it is invoked as *part* of the logicist reduction, so what I say above does not show this passage to be incompatible with Hylton’s reading. But my point is just that the passage fails to provide unambiguous

support for Hylton's interpretation because there is more than one way in which Peano's system might be invoked against Kant.

The second passage that can seem to provide support for Hylton's interpretation occurs in Russell's intellectual autobiography, *My Philosophical Development*. Russell reports that upon abandoning the Hegelianism of his undergraduate days he "began to believe everything the Hegelians disbelieved...Above all, I no longer had to think that mathematics is not quite true."⁶⁷ This remark suggests that Russell was indeed, as Hylton suggests, committed, in opposition to Hegelianism, to the absolute truth of mathematics. However, the question that concerns us is not whether Russell rejected the doctrine of partial truth—plainly he did—but rather whether his *logicism* plays an essential role in this rejection. Further examination of the relevant texts suggests that this is not so. Russell's argues against the doctrine of partial truth not by trying to establish the status of mathematics as absolutely true, but by attempting to undermine the neo-Hegelian's argument for the doctrine. According to Russell, this argument runs as follows:

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Assuming that we are not to distinguish between a thing and its "nature," it follows from the axiom [of internal relations, *viz.*, the assumption that "every relation is grounded in the natures of the related terms"⁶⁸] that nothing can be considered quite truly except in relation to the whole. If we consider "A is related to B," the A and the B are also related to everything else, and to say what the A and the B are would involve referring to everything else in the universe. When we consider merely that part of A's nature in virtue of which A is related to B, we are said to be considering A *qua* related to B; but this is an abstract and only partially true way of considering A, for A's nature, which is the same thing as A, contains the grounds of its relations to everything else as well as to B. Thus nothing quite true can be said about A short of taking account of the whole universe.⁶⁹ (*Philosophical Essays*, 140)

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Ignoring some of the details, we might charitably reconstruct Russell's reasoning as follows:

Everything in the universe is in some way related to everything else. By the axiom of internal relations, all these relations are grounded in intrinsic properties of their relata.⁷⁰ Consequently, ever so many intrinsic properties would have to be mentioned in any fully adequate intrinsic description of any entity. So, in practice, even intrinsic descriptions that appear exhaustive by ordinary standards will fall far short of adequacy. By the Hegelian doctrine that "abstraction"—which in this context means a partial consideration of something's nature—is falsification,⁷¹ most of our claims will therefore fall far

short of the highest degree of truth. Indeed, the only claims that will be “quite true” will be ones that contain an exhaustive description of the universe, claims, that is to say, about the world considered as a whole.

Evidently, this argument relies as much on the doctrine of abstraction as falsification as it does on the axiom of internal relations. Since these two premises are mutually independent, Russell’s portrayal of the latter principle as the *sole* ground of the other neo-Hegelian doctrines must be considered misleading.⁷² On the other hand, it is easy to see why Russell should have wanted to place so much weight on this so-called “axiom”; for he took the argument for philosophical monism to follow directly from it.⁷³

Russell rejects the axiom of internal relations,⁷⁴ and so the argument itself, because he recognizes that, although asymmetric relational facts—such as the fact that *a* is greater than *b*—cannot be reduced to facts about intrinsic qualities of their relata,⁷⁵ the acknowledgement of such facts as genuine aspects of reality is nonetheless required for the development of any satisfactory theory of order, and so for the formulation of any satisfactory philosophy of mathematics.⁷⁶

In addition to attacking the premises of the neo-Hegelian’s argument, Russell provides a set of more direct arguments against the thesis that “no proposition is quite true.” He argues, first, that this proposition is “sufficiently condemned by the fact that, if it be accepted, it also must be self-contradictory”;⁷⁷ second, that the neo-Hegelians cannot explain the difference between partly true and partly false propositions;⁷⁸ and, third, that they are unable to develop the consequences of their theory, because any deductions they might make from their partially true premises might rest on the false aspect of these premises and so be erroneous (*ibid*). I shall not evaluate these further criticisms, since the main point here is just that none of Russell’s arguments against the thesis of partial truth depend on logicism. That being so, the mere fact that Russell opposed this thesis cannot be taken to constitute support for Hylton’s reading.

6. Logicism and Epistemology

Because logicism is designed to explain among other things, the *certainty* of mathematics, it appears that epistemological considerations—contrary to Hylton—*do* play a role in motivating Russell’s project. Moreover, Russell does seem to concern himself with the question of the “source” of our knowledge of mathematics insofar as he characterizes the Kantian position he wishes to oppose in just

these terms. Kant, he says, “concluded that the propositions of mathematics all deal with something subjective, which he calls a form of intuition. Of these forms there are two, space and time, time is the *source* of Arithmetic, space of Geometry.”⁷⁹ (emphasis added).

The connection between logicism and epistemology may, in fact, be rather intimate. In the *Principles*, Russell says that: “The fact that all mathematical constants are logical constants, and that all the premises of mathematics are concerned with these, gives ... the precise statement of what philosophers have meant in asserting that mathematics is *a priori*.” (*Principles*, § 10). Since the alleged fact reported here is just the logicist thesis,⁸⁰ this remark suggests that to argue for logicism is thereby to defend the *a priori* status of mathematics. In one respect, of course, this remark is misleading. Logicism, after all, could not be taken to give the precise statement of what *Kant* means by calling mathematics “*a priori*”; so Russell must be speaking loosely here. He must mean that the logicist thesis amounts to a precise statement of what *some* philosophers have meant by asserting that mathematics is *a priori*, viz., that it is *a priori but not intuitively grounded*. This “precise statement” is supposed to be a clarification of the old epistemological claim, rather than a replacement of it, so, unless Russell wishes to dislodge the notion of the *a priori* from its traditional home in epistemology,⁸¹ we must conclude that for him logicism just *is* an epistemological thesis: it is the precise formulation of the thesis that mathematics has an *a priori* and non-intuitive status.

If Russell wishes to claim that mathematics is *a priori* and non-intuitive, why does he nonetheless deny that it is analytic?⁸² The answer, I believe, is that Russell is working with an extremely narrow conception of analyticity that derives from his literal-minded construal of (what is standardly taken to be⁸³) Kant’s most adequate formulation of an analyticity criterion. According to that (supposed) criterion, a judgement is analytic iff its truth “can be cognized sufficiently in accordance with the principle of contradiction.”⁸⁴ Russell, rather uncharitably, takes this to imply that to be analytic a judgement must follow from the law of contradiction *alone*. Thus, in his book on Leibniz, speaking of the “Law of Contradiction,”⁸⁵ Russell says:

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We may argue generally, from the statement of the Law of Contradiction, that no proposition can follow from it alone, except the proposition that there is truth, or that some proposition is true. ... Thus the doctrine of analytic propositions seems wholly mistaken. (*The Philosophy of Leibniz* (London: Routledge, 1992), 22.)

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Russell's point is that since almost nothing follows from the law of contradiction *alone*, almost everything is synthetic, and so the idea that there is an interesting analytic/synthetic distinction is mistaken.

One suspects that if Russell had been willing to interpret Kant as charitably as Frege did,⁸⁶ he might, after all, have concluded that mathematics was analytic.⁸⁷ And, interestingly, Russell did soon arrive at a conception of analyticity closer to Frege's. In his essay "Necessity and Possibility" of 1905, he says: "We may...usefully define as *analytic* those propositions which are deducible from the laws of logic,"⁸⁸ and he adds that to call pure mathematics "analytic" in this sense is an appropriate way of marking dissent from Kant on the question whether pure mathematics can be deduced from the laws of logic alone (ibid). It seems a plausible conjecture that Russell arrived at this more inclusive conception of analyticity as a result of reading Frege's *Grundlagen*, a work he mentions in his appendix to the *Principles* (§ 481). The upshot of these reflections is that it would be hasty to conclude from Russell's early lack of interest in the analytic/synthetic distinction that he found the question of the epistemological status of mathematics uninteresting or unimportant. All that does follow is that, early on, he would not have formulated the epistemological question in terms of the analytic/synthetic distinction.

7. Logicism, certainty, and the paradoxes

I have argued that the core motivation for logicism in the *Principles* is Russell's desire to develop an account of the nature of the object of mathematical knowledge that is compatible with the evident facts that mathematics is true and that it affords exact and certain knowledge. But why should showing mathematics to be a branch of logic serve to *explain* these features of our mathematical knowledge? In the case of the properties of truth and exactness, this is not hard to see: If we are prepared to accept that logic is true, then to exhibit the theorems of mathematics as theorems of logic will be to show them to be true also. Similarly, if the theorems of geometry are logical theorems, rather than theorems about actual space, then our knowledge of them will be unmediated by the senses, and its exactness, consequently, will not be limited by the accuracy of our measuring instruments. But why logicism should account for the *certainty* of mathematical knowledge is harder to say. The problem is that Russell could have hoped to offer logicism as an explanation of mathematical certainty only if he had regarded the axioms of logic as themselves certain, and yet this can seem doubtful.

Russell discovered the paradox that bears his name in the early summer of 1901, while he was still composing the *Principles*. Since he was later to say that the paradoxes showed that “intuition is not infallible,”⁸⁹ one might doubt whether during the composition of the *Principles* Russell could have continued to regard the apparent self-evidence of his axioms as a reliable sign of their truth. This doubt is reinforced by Russell’s talk around this time of “the sinister influence of obviousness,”⁹⁰ and also by his remark that “self-evidence is often a mere will-o’-the-wisp, which is sure to lead us astray if we take it as our guide.”⁹¹ Nonetheless, I think a plausible case can be made that the Russell of the *Principles* did indeed regard his axioms as reliably self-evident, and, furthermore, that he continued to do so perhaps as late as 1905. To appreciate this point we need to consider the context of Russell’s disparaging remarks about “self evidence” and “obviousness.” Russell’s mention of “the sinister influence of obviousness” occurs in the course of a discussion of work being done by followers of Peano. Russell describes the aim of this school as being “to discover the necessary and sufficient premises of the various branches of mathematics, and to deduce results (mostly known already) by a rigid formalism which leaves no opening for the sinister influence of obviousness.”⁹² Having made this remark, he continues:

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Those who know how many obvious mathematical propositions are false, how many highly paradoxical propositions are true, and how difficult it is, in verbal reasoning, to avoid unconscious employment of an obvious proposition, will appreciate the reasons for banishing words from our deductions, and effecting everything in a wholly symbolic language. (*Papers*, v. 3)

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Russell’s point in this passage, and in other places where he criticises self-evidence or obviousness during this period, is that principles which can seem obvious when formulated in *natural language* are apt to lose this appearance when expressed in an unambiguous symbolic notation. The remark about self-evidence’s being merely a “will-o’-the-wisp,” for example, is made in connection with the alleged self-evidence of the natural language claim that “a whole always has more terms than a part.”⁹³ When this statement is formulated as a claim about the cardinalities of sets, it loses any apparent obviousness it might otherwise derive from not being clearly distinguished from the obvious, yet tautological, claim that any set contains some terms not contained by its proper subset. Since Russell’s basic principles *are* formulated in symbolic notation, the remarks presently under consideration provide no

evidence that he regards the apparent self-evidence of his axioms as anything but a reliable guide to their truth.

Furthermore, Russell's writings in the period 1903–1905 contain several indications that he does indeed regard his axioms as self-evident. First, in § 18 he goes to great lengths to defend the self-evidence of his tenth axiom, which, he implies, has a deeper kind of self-evidence than such “more superficially obvious” propositions as the law of excluded middle (*ibid.*). Second, in his discussion of Kant's antinomies he is explicit that we *may* treat the self-evidence of a proposition as a guide to its truth, so long as it is not opposed by any other self-evident proposition (*Principles*, § 435). And, finally, as late as 1905 he objects to a candidate comprehension axiom on the grounds that it lacks “intrinsic plausibility.”⁹⁴

On my reading, there is a clear explanation of why Russell should have required self-evidence of his axioms in the *Principles*: if mathematics is to be certain, its fundamental basis must be certain, and so self-evident. On the other hand, if we suppose that upon discovering the paradoxes Russell immediately concluded that apparent self-evidence could no longer be regarded as evidence of truth, it is hard to see why he should have continued to place so much weight on the self-evidence of his premises. In my opinion, the most plausible explanation of Russell's attitude towards self-evidence at the time of the *Principles* is as follows: Although Russell appreciated that the lessons of the paradoxes—namely, that the apparent self-evidence even of principles formulated in a symbolic notation was not *always* to be trusted, he nonetheless continued to hope that he would eventually be able to draw a principled distinction between the unreliable self-evidence of naive comprehension and the reliable self-evidence of his axioms. That is to say, he continued to regard apparent self-evidence as a necessary condition on an acceptable axiom even while recognizing that it was not sufficient to justify its adoption. By 1906 he had clearly abandoned this hope,⁹⁵ but one suspects that his change of mind would have been prompted by his realization, in the light of his appreciation of the need for the non-self-evident axiom of reducibility,⁹⁶ that the requirement of self-evidence would, in any case, have to be jettisoned.

8. Logicism after 1905: the regressive method

In the years immediately following the publication of the *Principles* Russell came to appreciate that the logicist development of arithmetic would require axioms whose claim to be self-evident logical

truths was dubious or worse. In 1904 he recognized the need for a version of the axiom of choice—the so-called “multiplicative axiom.”⁹⁷ Two years later, he recognized the need for the axiom of infinity,⁹⁸ which he had previously regarded as derivable from logic alone (*Principles*, § 339). Russell was well aware of the lack of obviousness of these new axioms,⁹⁹ and he even suspected that the multiplicative axiom was false.¹⁰⁰ But, these developments did not prompt him to revise his requirement that the axioms of mathematics should be self-evident. Instead, Russell chose to treat the new “axioms” merely as hypotheses to be attached as antecedents to the theorems to be proved.¹⁰¹ We might feel that this maneuver changes the significance of Russell’s logicist thesis in a radical way—so much so, perhaps that Russell should no longer be regarded as a logicist.¹⁰² But *Russell* did not see things that way. Even after completing all three volumes of *Principia* he continued to maintain, in 1911, that “Pure mathematics ... can be expressed and proved entirely in terms of the ideas and axioms of logic.”¹⁰³ It appears that what changes in Russell’s view is not his commitment to logicism, but rather his conception of the true *content* of ordinary mathematical theorems. What mathematicians had previously regarded as the theorems of arithmetic would now have to be viewed as merely the consequents of the conditionals that constituted the true theorems.

A more momentous development than Russell’s discovery of the need for the so-called “axioms” of multiplicity and infinity was his adoption—c.a. 1906¹⁰⁴—of the axiom of reducibility. Russell treated this axiom not as an “hypothesis,” but as an axiom in its own right. Since this axiom is manifestly not self-evident, Russell had to adopt a new view of what makes a proposition acceptable as an axiom. He now maintained that an axiom is to be accepted either because it is itself self-evident, or because its consequences are.¹⁰⁵ In this way, he relegated intrinsic self-evidence from a requirement to a desideratum. Moreover, Russell now adopted the view that the fundamental “premises” of mathematics were less certain than some of its “conclusions.”¹⁰⁶ This radical revision of the order of knowledge in the foundational parts of mathematics led Russell to embrace a bold strategy, namely: to assimilate the methodology of foundational studies in mathematics to that of natural science:

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[The fundamental premises of mathematics] are too simple to be easy, and thus their consequences are generally easier than they are. Hence we tend to believe the premises because we can see that their consequences are true, instead of believing the consequences because we know the premises to be true. But the inferring of premises from consequences is the essence of induction; thus the method in

investigating the principles of mathematics is really an inductive method, and is substantially the same as the method of discovering general laws of any other science.¹⁰⁷

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Russell's abandonment of the requirement of self-evidence for the axioms of mathematics left logicism requiring a new rationale. If the basic principles of logic were not certain, logicism could not explain the certainty of mathematics.¹⁰⁸ This point was not lost on Russell, who now re-examined the question of logicism's purpose:

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If we are asked as to the use of [investigations conducted according to the new inductive method], several important uses may be mentioned. In the first place, when a number of facts are shown to follow from a few premises, this is not only a new truth in itself, but also an organisation of our knowledge, making it more manageable and more interesting. In the second place, the premises, when discovered, are pretty certain to lead to a number of new results which could not otherwise have been known.¹⁰⁹

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Logicism, then, is now considered useful, first, because it furnishes dependence results;¹¹⁰ secondly, because it amounts to a systematization of our knowledge, and, thirdly, because it involves the discovery of high-level principles from which the analogue of empirical predictions may be derived.¹¹¹ This last idea is stated in terms that leave no room for doubt that Russell takes the analogy between empirical and (foundational) mathematical method fully seriously:

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The law of gravitation...leads to many consequences which could not be discovered merely from the apparent motions of the heavenly bodies, which are our empirical premises. And so in arithmetic, taking the ordinary propositions of arithmetic as our empirical premises, we are led to a set of logical premises from which we can deduce Cantor's theory of the transfinite.¹¹²

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Positing general logical laws as true enables one to systematize the known mathematical facts, but it also enables one to discover *new* mathematical facts by deducing them from these laws.

Conclusion

This completes our investigation of Russell's reasons for logicism. We have found, on the positive side, that Russell's motivations are chiefly epistemological, mathematical and methodological in character, and, on the negative side, that a broad attack on idealist metaphysics is not among them. Further, by taking the widest possible view of Russell's motivations for logicism, we have uncovered a greater degree of continuity in Russell's view than is usually supposed. While Russell's adoption of the "regressive method" of discovering the principles of mathematics leads to the abandonment of the epistemic goal of explaining mathematical certainty, it does not mark a complete abandonment of his original logicist goals. The old themes of theoretical unification/organization of knowledge still loom large, as does the idea that axiomatics can be a spur to mathematical discovery—the inductive discovery of the premises, after all, is supposed to lead to the proving of new results "which could not otherwise be known."¹¹³ Moreover, logicism can still be viewed as a theory of the nature of mathematics—one that is, in contrast to idealism and empiricism, *compatible* with our common view of mathematics as body of *truths* of which *exact* knowledge is possible. It is plain that as late as 1911¹¹⁴ Russell continues to regard this as an advantage of logicism over its empiricist and Kantian competitors.¹¹⁵

¹ This thesis is propounded in *The Principles of Mathematics* [*Principles*] (first published, 1903, 2nd edition, (London: Allen and Unwin, 1996)), § 4. But its rigorous substantiation is not attempted until *Principia Mathematica* [*Principia*], Bertrand Russell and Alfred North Whitehead (Cambridge: Cambridge University Press), 1910–13, second edition 1925. For other statements of the thesis see *Logical and Philosophical Papers 1909–13*, *The Collected Papers of Bertrand Russell, Volume 6* [*Papers*, v. 6], edited by John G. Slater with the assistance of B. Frohmann (London: Routledge, 1992), 43, and Bertrand Russell, *My Philosophical Development* (London: Routledge, 1993), 57.

² See Nicholas Griffin, “Russell on the Nature of Logic (1903–1913),” *Synthese*, 45, no. 1 (1980), 117–188. Griffin’s suggestion is discussed in note 108 below.

³ See Peter Hylton, “Logic in Russell’s Logicism,” in David Bell and Neil Cooper, eds., *The Analytic Tradition* (Oxford: Blackwell, 1990) and Peter Hylton, *Russell, Idealism and the Emergence of Analytic Philosophy* [*Russell, Idealism*] (Oxford: Oxford University Press, 1990). Hylton’s suggestion is discussed below in § 5.

⁴ Paul Arthur, Schilpp, ed., *The Philosophy of Bertrand Russell* [*Russell*] (La Salle, Illinois: Open Court, 1989), 13; cf. Bertrand Russell, *Toward the “Principles of Mathematics,” 1900–02*, *The Collected Papers of Bertrand Russell, Volume 3* [*Papers*, v. 3], Gregory H. Moore, ed. (London: Routledge, 1993), 57.

⁵ Versions of the tendency to emphasize opposition to Kant as a motivation for Russell’s logicism at the expense of other considerations can be found in philosophers as far apart on other issues as J. Alberto Coffa and Peter Hylton. See J. Alberto Coffa, “Russell and Kant,” *Synthese*, 46 (1981): 247–263, Hylton “Logic in Russell’s Logicism,” and Hylton, *Russell, Idealism*, ch. 5.

⁶ See *Principles*, § 6 and D. Lackey, ed., *Essays in Analysis* [*Essays*] (London: George Allen and Unwin Ltd., 1973), 301.

⁷ *Principles*, second introduction, vii.

⁸ *Ibid.*

⁹ A third aspect of Russell’s logicism about geometry is the philosophical thesis that the true theorems of (what we should call pure geometry) are the conditionals one obtains by defining in purely logical terms all the non-logical vocabulary in the conditionals of the form described in (i). But this is not a point we need emphasize in the present context.

¹⁰ For this suggestion see, for example, Coffa, “Russell and Kant,” 252.

¹¹ For details of this strategy, and of the philosophical problems to which it gives rise, see Robert Hale, “Structuralism’s Unpaid Epistemological Debts,” *Philosophia Mathematica*, 3 (1996): 124–147—a paper to which my formulations of Russell’s “pure structuralist” position are indebted.

¹² Note that since Russell’s logicism involves both if-thenism and definitional logicism there are two respects in which it presupposes analysis.

¹³ *Principles*, § 423; cf. *Papers* v. 3, 378.

¹⁴ Here it pays to recall that in the *Principles* Russell is just as much an if-thenist about arithmetic as he is about geometry, but of a radically different (and less interesting) kind. (See § 1 above).

¹⁵ Since Russell is happy to move freely between discussing the validity of an argument and the logical truth of a conditional, we may take him to tacitly presuppose the deduction theorem—or less charitably, to have failed to notice the need to recognize it as an isolable assumption in his reasoning.

¹⁶ To say this is not, of course, to deny that Russell planned to establish that the axioms of arithmetic are theorems of logic. It is just to observe that in the case of arithmetic, as opposed to geometry, Russell did not feel that he needed to combat the idea that there are mathematically valid, but logically invalid, modes of reasoning—a view defended by those who held diagrammatic reasoning to be essential to (some) geometrical proofs.

¹⁷ See Paul Guyer and Allen Wood, trans. and eds., *The Cambridge Edition of the Works of Immanuel Kant, Critique of Pure Reason [Critique]* (Cambridge: Cambridge University Press, 1998), A 163–4/B 204–5, and “Letter to Johann Schultz,” *The Cambridge Edition of the Works of Immanuel Kant, Correspondence*, translated and edited by Arnulf Zweig (Cambridge: Cambridge University Press, 1999), 284.

¹⁸ In the *Inaugural Dissertation* Kant remarks that “PURE MATHEMATICS deals with *space* in GEOMETRY, and *time* in pure MECHANICS.” He goes on to say that number is the concept treated of in arithmetic. *The Cambridge Edition of the Works of Immanuel Kant, Theoretical Philosophy, 1755–1770*, translated and edited by David Walford in collaboration with Ralf Meerbote (Cambridge: Cambridge University Press, 1992), 390. So arithmetic is not the science of time, as geometry is the

science of space. A helpful discussion of this point is contained in Michael Friedman, *Kant and the Exact Sciences* (Cambridge Mass: Harvard University Press, 1992), ch. 2, § 2. But the issues here are complex. One might argue that although arithmetic is not *about* time, nevertheless it *depends* on time. For a defence of this view, see Robert Hanna, “Mathematics for Humans: Kant’s Philosophy of Arithmetic Revisited,” *European Journal of Philosophy*, 10 (2002): 328–353.

¹⁹ *Principles*, § 423; cf. *Papers* v. 3, 378.

²⁰ *The Problems of Philosophy*, first published 1912 (Oxford: Oxford University Press, 1986), 49.

²¹ The article was written in 1902 but not published until 1907, when it appeared in the *New Quarterly*.

²² *Papers*, v. 3, 366.

²³ Quoted in *Philosophical Papers 1896–99, The Collected Papers of Bertrand Russell, Volume 2* [*Papers*, v. 2], edited by Nicholas Griffin, Albert C. Lewis, and William C. Stratton (London: Routledge, 1990), 160, cf. 163. Cf. Schilpp, *Russell*, 14.

²⁴ Letter to Jourdain, 10 April, 1910, quoted in I. Grattan-Guinness, *Dear Russell, dear Jourdain* [*Dear Russell*] (Duckworth, 1977), 133. It is noteworthy, in light of this remark, that Russell’s first informal statement of logicism does not occur until January 1901. See *Papers*, v. 3, 367.

²⁵ *Mysticism and Logic*, 54 and 56; cf. *Papers*, v. 3, 358.

²⁶ *An Essay on the Foundations of Geometry* (New York: Dover, 1956), §§ 56 and 60.

²⁷ *Ibid.*, § 10.

²⁸ *Principles*, § 352.

²⁹ Cf. *Papers*, v. 6, 51. Strictly, Russell would at best be entitled to claim that it provided evidence of the unprovability in the system in question of what one was trying to prove. However, Russell does not seem to be sensitive to this distinction.

³⁰ *Papers*, v. 3, 368.

³¹ A more celebrated example of this phenomenon is Cantor’s attempt to prove the result, in his day supposed “self-evident,” that the two-dimensional continuum cannot be mapped one-one onto the linear continuum. Three years of failed attempts to establish this result led Cantor to ask whether the mapping whose existence he was trying to disprove might after all exist. Cantor’s discovery that indeed it did prompted his famous remark: “I see it, but I don’t believe it.” The result is given in Georg

Cantor, “Ein Beitrag zur Mannigfaltigkeitslehre,” *Journal für die reine und angewandte Mathematik*, French translation in *Acta Mathematica* 2 (1878). For further discussion of this example see A. A. Fraenkel, *Abstract Set Theory*, 2nd edition (Amsterdam, North-Holland Publishing Company, 1961). Russell knew of the result and mentions it in an early draft of the *Principles* (*Papers* v. 3, 152), but whether he was familiar with the history of its discovery is not clear.

³² Cf. *Papers* v. 3, 383. Relevant here are Russell’s comments on his own unwitting use of the multiplicative axiom, and his subsequent discovery that it was an independent axiom (*Papers*, v. 6, 50).

³³ *Papers*, v. 3, 377–8; cf. *Mysticism and Logic*, 72.

³⁴ *Principles*, § 389.

³⁵ Cf. Hylton, “Logic in Russell’s Logicism.”

³⁶ *Ibid.*, 144; cf. Hylton, *Russell, Idealism*, 168.

³⁷ Hylton, “Logic in Russell’s Logicism,” 145.

³⁸ *Ibid.*, 143.

³⁹ Elsewhere Hylton describes this claim as “a necessary part of any form of idealism” (*Russell, Idealism*, 181).

⁴⁰ “Logic in Russell’s Logicism,” 140.

⁴¹ Hylton, *Russell, Idealism*, 180.

⁴² Hylton, “Logic in Russell’s Logicism,” 144. Here one gets the impression that Hylton is inviting us to think of Kant, Hegel, and the British neo-Hegelians, but he does not say so explicitly.

⁴³ *Russell, Idealism*, 180, emphasis added. Of course, logicism could not have motivated Russell’s own anti-idealism since Russell had already begun to move away from idealism by 1898–9 (see *My Philosophical Development*, 48, and “The Classification of Relations,” *Papers*, v. 2, 138–46) and, as we have seen, Russell’s logicism was only made possible by his study of Peano’s work during the summer of 1900—and not enunciated until January 1901 (*Papers*, v. 3, 367). But, I take it that Hylton’s idea is that Russell conceived of logicism as a means of winning new converts to his (and Moore’s) anti-idealism.

⁴⁴ See, for example, *Russell, Idealism*, 180.

⁴⁵ They must also accept that unconditional truth is preserved by logical inference, but, for the sake of argument, I will suppose that the attribution of this view to idealists is reasonable.

⁴⁶ Francis H. Bradley, *Appearance and Reality: a Metaphysical Essay* [*Appearance and Reality*], second edition with appendices [1897] (Oxford: Clarendon Press, 1968), 328.

⁴⁷ Russell read *Appearance and Reality* in 1894 and again in 1897. One supposes that, second time around, he would have read the second edition, which appeared that year, but this is not known for sure.

⁴⁸ “On the Nature of Truth,” *Proceedings of the Aristotelian Society*, n.s. 7 (1906–7). The first two sections of this article appeared as chapter 6 of Russell’s *Philosophical Essays* (London: Routledge, 1994), under the title “The Monistic Theory of Truth.”

⁴⁹ Harold Joachim, *The Nature of Truth* [*Truth*] (Oxford: Clarendon Press, 1906), 113.

⁵⁰ Cf. Russell, *Philosophical Essays*, 131, fn. 2.

⁵¹ Joachim, *Truth*, 113.

⁵² Russell, *Philosophical Essays*, 132.

⁵³ *Principles*, § 3, cf. xviii.

⁵⁴ On this point, see the remarks about asymmetric relations in section 5 below.

⁵⁵ “Logic in Russell’s Logicism,” 142–143.

⁵⁶ *Russell, Idealism etc.*, 181.

⁵⁷ *Ibid.*, cf., Hylton, “Logic in Russell’s Logicism,” 143.

⁵⁸ Hylton, “Logic in Russell’s Logicism,” 143.

⁵⁹ Since Russell sees Kant as maintaining that arithmetic, broadly construed, rests on time (*Principles*, § 433), this is the most promising case for generating a circularity of the kind Hylton considers.

⁶⁰ The cardinality of the set A = the cardinality of the set B iff A is equinumerous to B .

⁶¹ *Papers*, v. 3, 363–379.

⁶² *Papers*, v. 3, 369.

⁶³ *Papers*, v. 3, 350–362.

⁶⁴ *Papers*, v. 3, 359. Recall that for Russell this means what we should call “logical *cum* set theoretic” terms.

⁶⁵ Ibid.

⁶⁶ *Papers*, v. 3, 368.

⁶⁷ *My Philosophical Development*, 49. Whether neo-Hegelians besides the young Russell himself ever subscribed to such an “axiom” is open to question. On this point see Nicholas Griffin, *Russell’s Idealist Apprenticeship* (Oxford: Oxford University Press, 1991), 320–321.

⁶⁸ See Russell, *Philosophical Essays*, 139.

⁶⁹ Ibid., 140.

⁷⁰ Ibid., 139.

⁷¹ For Russell’s awareness of this doctrine see: *The Collected Papers of Bertrand Russell, Volume 1, Cambridge Essays, 1888–99* [*Papers*, v. 1], Kenneth Blackwell, Andrew Brink, Nicholas Griffin, Richard A Rempel, John G. Slater eds. (London: George Allen and Unwin Ltd.), 121, Russell, *My Philosophical Development*, 32, and *Principles*, §§ 134, 439.)

⁷² Russell, *My Philosophical Development*, 49.

⁷³ As Russell sees it, the argument for monism runs as follows: By the axiom of internal relations, all relations are “reducible to adjectives of their terms.” In other words, facts apparently of the form “ aRb ” are really of the form “ $r_1 a$ and $r_2 b$.” But the relation of non-identity cannot be handled in this way. So statements apparently of the form “ $a \neq b$ ” express no facts. So there is no diversity. Q. E. D. (For full details of the argument see Russell, *Philosophical Essays*, 141–2.) Russell does not discuss the obvious problem that one could equally reason from the fact that identity statements cannot be “reduced to adjectives of their terms,” that statements apparently of the form $a=b$ express no facts, and conclude that there is no unity.

⁷⁴ The principle of abstraction as falsification, for its part, is rejected without argument (*Principles*, §§138, 439).

⁷⁵ We might begin by saying that “ a is greater than b ” means “ a has magnitude m and b has magnitude n ,” but we will need to add “and m is greater than n ,” and this clause will itself stand in need of further analysis. Since Russell rejects the idea that any proposition could have an infinite analysis (*Principles* § 55), at least one instance of “ x is greater than y ” will therefore have to be acknowledged as unanalysable, or as Russell puts it, “ultimate” (*Philosophical Essays*, 144; *Principles*, § 214).

⁷⁶ Cf. *Principles*, § 216.

⁷⁷ *Ibid.*, § 215.

⁷⁸ Russell, *Philosophical Essays*, 139.

⁷⁹ *Principles*, § 433.

⁸⁰ Cf. *Principles*, § 412.

⁸¹ Evidence that Russell did indeed see the *a priori* as an epistemological category may be found in his 1905 essay “Necessity and Possibility” in *Foundations of Logic 1903–05, The Collected Papers of Bertrand Russell, Volume 4*, edited by Alasdair Urquhart with the assistance of Albert C. Lewis (London: Routledge, 1994), 508–523). Russell says: “The terms *a priori* and *empirical* seem to be more or less connected with the terms *necessary* and *contingent*, and to differ chiefly by the fact that they belong to theory of knowledge rather than to logic.” (*ibid.*, 509–10).

⁸² *Principles*, § 434.

⁸³ For doubts about whether Kant really intended this formulation as a criterion of analyticity see Ian Proops, “Kant’s Conception of Analytic Judgment,” *Philosophy and Phenomenological Research*, Vol. 70, No. 3 (May 2005): 588–612.

⁸⁴ *Ibid.*, A 151/B190.

⁸⁵ Russell actually states the principle *tertium non datur*.

⁸⁶ Cf. Gottlob Frege, *Die Grundlagen der Arithmetik: eine logisch-mathematische Untersuchung über den Begriff der Zahl*, Christian Thiel, ed. (Hamburg: Meiner, 1996), §§ 3 and 88.

⁸⁷ Frege emphasizes Kant’s second criterion, and interprets it to mean that an analytic truth is one that follows from the laws of logic and definitions. Whether such latitude really amounts to an explication of Kantian analyticity as opposed to a *redefinition* of it is a good question, but one beyond the scope of the present inquiry.

⁸⁸ *Papers*, v. 4, 516.

⁸⁹ Lackey, *Essays*, 194.

⁹⁰ *Papers* v. 3, 352.

⁹¹ *Ibid.*, 368.

⁹² *Ibid.*, 352.

⁹³ *Ibid.*, 368.

⁹⁴ Lackey, *Essays*, 147.

⁹⁵ Ibid., 194.

⁹⁶ Ibid., 281.

⁹⁷ *Papers*, v. 6, 50.

⁹⁸ Lackey, *Essay*, 15.

⁹⁹ *Papers*, v. 6, 43.

¹⁰⁰ Ibid., 51.

¹⁰¹ Letter to Jourdain 1st June, 1907, quoted in Grattan-Guinness, *Dear Russell*, 105–6.

¹⁰² For a forceful statement of this view, see George Boolos, “The Advantages of Honest Toil over Theft,” in *Logic, Logic, and Logic* (Cambridge Mass: Harvard University Press, 1998): 255–274.

¹⁰³ *Papers*, v. 6, 43.

¹⁰⁴ The axiom is first stated in Russell’s 1906 paper “On Insolubilia” (Lackey, *Essays*, 212).

¹⁰⁵ Ibid., 194; cf. *Principia*, 59.

¹⁰⁶ Lackey, *Essays*, 272.

¹⁰⁷ Ibid., 273–4; cf. 194.

¹⁰⁸ This point is enough to cast doubt on Nicholas Griffin’s interpretation of Russell’s motivations for logicism. Griffin argues that c.a. 1903 Russell’s logicism is intended to demonstrate the necessity of the propositions of mathematics—by demonstrating that its propositions can be derived from necessary axioms, but that c.a. 1908–1913 it is intended to demonstrate their certainty (Griffin, “Russell on the Nature of Logic (1903–1913),” 118). I take the latter claim to be refuted by the present point. The former runs into difficulties on another score. For Griffin the relevant conception of necessity is the one Russell outlines in his 1905 paper “Necessity and Possibility.” According to (a cleaned up version of) this conception, a proposition is necessary just in case it is an instance of a propositional function all of whose values are true. Griffin wrestles with certain problems for this account, but he does not mention the following difficulty. The propositions “ \supset is a relation,” and “ \in is a relation,” which in 1903 Russell regards as axioms of logic, are not necessary according to this definition, since “Bush is a relation,” for example, which is an instance of the propositional function “ \hat{u} is a relation,” is false. So in 1903 Russell could not have held the axiomatic basis of mathematics to be necessary in this sense.

That said, we may still ask whether the graded notion of necessity Russell offers in the *Principles*—which, given its date, would, in any case, seem to be the more natural notion to appeal to in attributing views to Russell c.a. 1903—affords a notion according to which the axioms of logic are necessary. And the answer is “yes.” Russell says: “A proposition is more or less necessary according as the class of propositions for which it is a premiss is greater or smaller. In this sense the propositions of logic have the greatest necessity, and those of geometry have a high degree of necessity” (*Principles*, § 430). This account, however, can be of no use to Griffin, for we cannot take the necessity of the axioms to be transmitted from premises to conclusions, since on the present account derived principles have less necessity than axioms.

¹⁰⁹ Lackey, *Essays*, 282–3; cf. 195.

¹¹⁰ *Ibid.*, 272.

¹¹¹ Russell goes on to mention a further, vaguer, motivation in addition to these, but it is hard to see what he has in mind.

¹¹² Lackey, *Essays*, 275.

¹¹³ *Ibid.*, 282–3.

¹¹⁴ *Papers* v. 6, 136–137.

¹¹⁵ My thanks to the following philosophers for comments on earlier drafts: Jamie Tappenden, Peter Sullivan, Michael Potter, Nicholas Griffin, Gideon Makin, Jim Joyce, Bernard Linsky, Jason Bridges, Jessica Wilson, Benjamin Hellie and an anonymous referee for the Journal. A distant ancestor of the paper was presented to The Cambridge University Moral Sciences Club in October, 2001. I am grateful to the audience on that occasion for their helpful questions and comments. My thanks, also, to Thomas Ricketts for encouraging me to examine these issues.