Monopoly versus Competition in Banking: Some Implications for Growth and Welfare

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Abstract

We consider how the industrial organization of the financial system affects an economy's macroeconomic performance. In particular, we compare two otherwise identical monetary economies — one with a competitive and the other with a monopolistic banking system — along the dimensions of bank liquidity provision, asset allocation, savings behavior, long-run real growth, and depositor welfare. We find that monopoly in banking can potentially be growth promoting. However, the relative performance of the two banking systems depends crucially on the level of nominal interest rate.

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1 Introduction

It is now empirically well established that there are strong connections between various aspects of an economy's financial system, and the economy's long-run real performance. For instance, King and Levine (1993a, b) and Levine, Loyaza, and Beck (2000) show that long-run rates of real growth are very strongly correlated with measures of bank lending to the private sector, and with other measures of banking activity. King and Levine (1993a, b) go so far as to argue that measures of banking system development are the only robustly significant predictors of an economy's future growth performance, and Levine, Loyaza, and Beck (2000) provide evidence suggesting that the direction of causation in the data is from financial market development to real development.

There is also a large theoretical literature — originating with Bencivenga and Smith (1991) and Greenwood and Jovanovic (1990) — that elaborates on various mechanisms by which the development of financial intermediation can enhance growth. Following the suggestions of Bagehot (1873) and Hicks (1969), Bencivenga and Smith (1991) focus on how bank liquidity provision affects the allocation of wealth and, through this channel, the amount of capital investment in an economy. Greenwood and Jovanovic (1990) emphasize the role of banks in eliciting information about the relative productivity of various investments, so that banks affect an economy's portfolio allocation in a way that promotes growth. The key feature of both strands of the literature is that they bring into the foreground the issue of how banks affect the way in which agents hold their wealth.

Virtually the entire theoretical literature on financial markets and growth considers economies with competitive banking systems. Little attention is devoted to whether the industrial organization of the banking system matters for growth, or for anything else of macroeconomic consequence. There is also virtually no empirical work on this topic. This paper is a first attempt to fill this gap in the literature.

Specifically, we compare two economies that are identical in all respects except for the degree of market power enjoyed by their banks. In one economy the banking system is competitive. In the other it is monopolistic. We then ask how the industrial organization of the banking system affects bank liquidity provision, the allocation of assets, savings behavior, and long-run real growth rates. We also compare a measure of depositor welfare under the two systems.

Interestingly, it turns out that little can be said about the relative performance of competitive versus monopolistic banking systems without knowing something about monetary policy, and particularly, nominal rates of interest. The reason for this interconnection between monetary policy and the relative performance of different banking systems is related to the function that banks perform in our model.

We consider an economy in which banks provide liquidity. In particular, bank depositors face some risk of needing to liquidate capital investments in favor of cash. If capital yields a higher return than cash, i.e. if nominal rates of interest are positive, then agents will want to be insured against this risk. This insurance can be provided by banks resembling those in Diamond and Dybvig (1983). Banks take deposits and allocate the funds between the two primary assets. They offer agents the option of withdrawing their funds early (at a penalty), should they experience a liquidity need. A monopolistic bank differs from a competitive one in the return structure if offers to depositors and in the way it allocates assets.

The difference between the behavior of a monopolist versus a competitive bank depends on the return differential between capital and cash — the nominal interest rate. When nominal rates of interest are very low, cash becomes a good asset. When nominal rates of interest are zero, cash is as good an asset as anything else; agents can avoid banks altogether if they so desire and can use cash for all transactions. As a result, in environments with low nominal rates of interest, monopoly banks have little market power and competitive and monopolistic banking systems behave very similarly. However, as nominal rates of interest rise, demand for insurance rises and monopoly banks will enjoy a greater degree of monopoly power. What macroeconomic consequences result?

For a given nominal interest rate, either a competitive or a monopolistic banking system can lead to higher rates of real growth. Competitive banks earn no profits. Monopoly banks earn profits from investments in high yielding capital, but not from holding cash reserves. Hence a monopolized banking system holds a lower fraction of its assets in cash reserves than a competitive banking system, and a larger fraction of its assets in the form of capital investments. By itself, this would tend to lead to higher real rates of growth in a monopolistic than in a competitive banking system. However, monopoly banks also depress the rates of return received by depositors. In the absence of strong income effects, the result is that an economy with a monopoly in banking has a lower savings rate than an economy with a competitive banking system. The lower savings rate tends to depress growth. Which effect dominates — the effect of monopoly banks on asset allocations or their effect on savings behavior — depends primarily on two factors: the level and interest elasticity of savings, and the nominal rate of interest. For economies with high and relatively interest inelastic savings, the effect on portfolio composition dominates, and real growth rates will be higher with a monopoly in banking. For economies with low and relatively interest elastic savings rates, the effect on savings dominates, and a monopoly in banking will be detrimental to growth.

In addition, the level of nominal interest rates matters because this level affects the market power of a monopoly bank. When nominal interest rates are low, even a monopoly bank has little market power. Hence monopoly in banking has little or no effect on either the composition of aggregate assets, or on aggregate savings behavior. Or, in other words, in environments with low nominal interest rates, the industrial organization of the banking system has minimal consequences for growth.

In terms of welfare, a competitive banking system always provides liquidity (risk-sharing) in a way that is at least weakly superior to that of a monopolistic banking system. If real growth rates are higher under competition than monopoly, all bank depositors will prefer that the banking system be competitive. However, if a monopoly in banking is growth promoting, society will perceive a trade-off between higher growth and better risk-sharing. We describe some conditions under which either type of banking system will be socially preferred. Again, in economies with sufficiently low nominal rates of interest, the choice between competitive versus monopolistic banking systems is irrelevant from a welfare perspective. Perhaps surprisingly, however, this choice is also irrelevant in economies with very high nominal rates of interest (rates of inflation). Thus the inflationary environment can very much affect the relative rankings of competitive versus monopolistic banking systems.

Our vehicle for examining these issues is an endogenous growth model [1] that incorporates money and banks. Spatial separation and limited communication creates a transactions role for money, as in Townsend (1980, 1987). The risk of premature asset liquidation creates a role for banks, as in Diamond and Dybvig (1983). The model actually used closely resembles the models of Champ, Smith and Williamson (1996), Schreft and Smith (1997, 1998), or Paal and Smith (2002).

[1] It is of no importance to our results that we employ an endogenous growth model. Analogous results would emerge for steady states in a version of our model that shuts down the potential for permanent growth.

The remainder of the paper proceeds as follows. Section 2 lays out the general environment we analyze. Section 3 considers three different mechanisms by which assets can be traded – without any financial intermediary, with competitive banks and with a monopolistic bank. Section 4 compares real rates of growth with competitive versus monopolistic banking systems in a general equilibrium framework, while section 5 undertakes a welfare analysis of the industrial organization of the banking system. Proofs of selected lemmas and propositions are given in the Appendix.

2 The Environment

We consider a discrete time economy, with time indexed by $t = 0, 1, 2, ...$ The economy consists of two locations, or islands. On each island at each date a new set of two-period-lived agents are born. Within each generation, there are two types of agents. One type we will refer to as depositors. There are a continuum of these agents with unit mass[2]. All depositors are identical ex ante (but not ex post). In addition, each generation on each island contains N bankers. Setting $N = 1$ yields a monopolistic banking system while setting $N > 1$ yields a competitive one.

At each date a single final good is produced using capital and labor as inputs. Let K_t denote the time t capital stock of a typical firm, let L_t denote the firm's labor input, and let $\bar{k_t}$ denote the aggregate capital-labor ratio. Since we wish to allow for endogenous growth in a simple way, we follow Shell (1966) and Romer (1986) and assume that the output of a representative producer at t is

$$F(K_t, L_t, \bar{k_t}) = A{K_t}^{\alpha}{L_t}^{1-\alpha}\bar{k_t}^{1-\alpha},$$

with $\alpha \in (0, 1)$. Each individual producer takes $\bar{k_t}$ as given. The capital

[2]We thus abstract from population growth. This is with no real loss of generality.

stock depreciates at the rate $\delta \in [0,1]$.

All young depositors are endowed with a single unit of labor, which they supply inelastically. They have no other endowments of goods, money, labor, or capital at any date. Let c_{1t} and c_{2t}^j denote the first and second period consumption of a young depositor. Here the superscript j stands for agent type; $j = m$ for "movers" and $j = n$ for "non-movers". We will discuss different agent types shortly. For now it is sufficient to note that agents learn their types at the beginning of the second period of their lives and the probability of becoming a mover is π. Then this agent has the lifetime expected utility

$$u(c_{1t}, c_{2t}) = \theta \left[\frac{c_{1t}^{1-\rho}}{1-\rho} \right] + \left[\pi \frac{(c_{2t}^m)^{1-\rho}}{1-\rho} + (1-\pi) \frac{(c_{2t}^n)^{1-\rho}}{1-\rho} \right], \tag{2.1}$$

with $\rho \in (0,1)$.

Bankers, on the other hand, have no endowments, care only about second period consumption, and are risk-neutral.

The economy has two primary assets: physical capital and fiat money. One unit of the final good produced but not consumed at t yields one unit of capital at $t+1$. Undepreciated capital can either be retained as capital, or converted into consumption. With respect to money, let M_t denote the stock of currency outstanding, per depositor, at the end of period t. We assume the nominal money stock grows at the exogenously set gross rate σ. Thus $M_{t+1} = \sigma M_t$, with $M_{-1} > 0$ given as an initial condition.

At $t = 0$ there also exist a generation of initial old with unit mass on each island. They own the initial money stock M_{-1} and the initial capital stock K_0.

Finally, we assume (for maximum simplicity) that money creation funds an endogenously determined sequence of government expenditures. These expenditures do not affect agents' savings behavior or portfolio allocations.

Discussion

To this point we have made three assumptions whose role should be discussed. The first is that there is an externality in production that allows endogenous growth to occur. We employ this formulation for simplicity only; the production externality has *no* meaningful consequences for our results. At some cost in additional complexity, we can obtain exact analogs of all of our results for the steady states of economies that have no externality in production.

The second assumption is that $\rho \in (0,1)$. If $\rho > 1$, as in standard overlapping generations economies, income effects are "large". This will have unintuitive consequences. In particular, reserve-holdings by banks will be increasing with the nominal interest rate (the opportunity cost of holding reserves). This is not the empirically plausible case (Schreft and Smith (2002)). We have opted *not* to focus on the consequences of large income effects.

The third assumption is that money creation is used to finance an endogenous sequence of government expenditures. Again, this formulation is intended only to maximize simplicity. It is conceptually straightforward to have monetary injections/withdrawals accomplished through lump-sum tax/transfers made to young depositors. A formulation of this type (for a competitive banking system) appears in Paal and Smith (2002).

2.1 Spatial Separation, Limited Communication, and Liquidity Preference Shocks

Following Townsend (1980, 1987), we introduce a transactions role for currency by emphasizing the spatial separation of — and the limited communication between — distinct markets. Thus we assume that, in each period, agents can trade and communicate only with other agents who inhabit the same location. We also introduce a role for banks by allowing agents to face "liquidity preference shocks", which take the following form. At date t, a young depositor faces the probability π that he will be (exogenously) forced to move to the other location between t and $t+1$. In the absence of inter-location communication, relocated depositors cannot transact with credit instruments (checks, credit cards) in their new location. Moreover, capital investments cannot be transferred between locations. Therefore, agents who are relocated require currency to transact. Moreover, a depositor who is randomly relocated is forced to convert other, potentially higher yielding, assets into cash. This represents an adverse shock, against which depositors will wish to be insured. As in Diamond and Dybvig (1983)[3], this insurance can be provided through banks. The behavior of banks is described below. Finally, we assume that bankers are not relocated, so that they can always be contacted by their depositors. This is, in fact, what allows them to function as banks.

The timing of events within a period is as follows. First, firms rent capital and labor, produce the final good, and pay their factors of production. Final goods are then either consumed or are invested to create next period's capital stock. In particular, young depositors choose how much to save. This savings may or may not be intermediated, so that depositors can either hold the economy's primary assets directly or indirectly. In either case, portfolios are allocated between currency and capital. After this portfolio allocation occurs, the specific identities of the depositors who are to be relocated are revealed. These agents then convert other assets into currency and move to their new location. They use the currency obtained to finance consumption at $t+1$. Agents who are not relocated do not require currency to make purchases, as there is complete within-location communication. Thus relocation constitutes a physical story about which transactions require cash and which do not. This timing of events is depicted in Figure 1.

[3] See also Greenwood and Smith (1997).

2.2 Goods and Factor Markets

In the next section, we will consider asset trading in more details. The behavior of goods and factor markets is independent of how assets are traded. At each date and in each location, firms rent capital and labor in competitive factor markets. Thus all factors are paid their marginal products. Let w_t denote the time t real wage and r_t denote the time t capital rental rate. Then

$$r_t = F_1(K_t, L_t, \bar{k}_t) = \alpha A \tag{2.2}$$

$$w_t = F_2(K_t, L_t, \bar{k}_t) = (1-\alpha)Ak_t \equiv w(k_t) \tag{2.3}$$

hold, where equations (2.2) and (2.3) both exploit the fact that, in equilibrium, $k_t = \bar{k}_t$. The gross real rate of return on capital investments between t and $t+1$, R_t, is

$$R_t = \alpha A + 1 - \delta \equiv R. \tag{2.4}$$

Final goods are also sold in competitive markets. We let p_t denote the dollar price at t of a unit of the final good. The gross real rate of return on cash balances between t and $t+1$ is p_t/p_{t+1}.

3 Financial Intermediation and Asset Trading

This section describes in more details how assets are traded under different financial intermediation mechanisms. First, we assume that banks do not operate (or agents do not use them), so that depositors hold and trade primary assets directly. Second, we consider a competitive banking system ($N > 1$). Third, we examine monopoly in banking ($N = 1$).

3.1 Unintermediated Asset Trade

After being paid w_t at date t, young depositors choose how much to save, and how to allocate their savings between direct holdings of capital investments and currency. Once these savings/portfolio allocations have occurred, each depositor learns whether or not he is to be relocated. Depositors then meet in a market where relocated agents sell their capital investments to non-relocated agents for cash. Relocated agents then take the cash acquired to their new location, and use it to purchase consumption when old.

Let $s_t = w_t - c_{1t}$ be the savings of a young depositor at t, let i_t be the amount of his capital investment and let m_t be the amount of his real money holdings. Let q_t be the dollar price of a unit of capital in the post-relocation capital resale market. Let $i_t'^j$ and $m_t'^j$ denote the capital and real money holdings of a young depositor *after* having traded at the capital resale market, where $j = m, n$ signifies agent type, as before. With this, the constraints that a young agent faces can be formulated as:

$$\begin{aligned} c_{1t} &\leqq w_t - s_t \\ i_t + m_t &\leqq s_t \\ i_t'^j + \frac{p_t m_t'^j}{q_t} &\leqq i_t + \frac{p_t m_t}{q_t} \\ c_{2t}^m &\leqq \frac{p_t m_t'^m}{p_{t+1}} \\ c_{2t}^n &\leqq i_t'^n R + \frac{p_t m_t'^n}{p_{t+1}} \end{aligned} \tag{3.1}$$

A young agent of generation t chooses c_{1t}, s_t, i_t, m_t, $\left\{i_t'^j, m_t'^j, c_{2t}^j\right\}_{j=m,n}$ to maximize (2.1), subject to the constraints (3.1) and non-negativity constraints on consumption and asset holdings. Let the optimal value of each choice variable x_t in this problem be denoted by $\tilde{x}_t$. Then equilibrium in the capital resale market requires

$$\pi \tilde{\imath}_t^{\prime m} + (1-\pi)\, \tilde{\imath}_t^{\prime n} = \tilde{\imath}_t$$

and

$$\pi \tilde{m}_t^{\prime m} + (1-\pi)\, \tilde{m}_t^{\prime n} = \tilde{m}_t.$$

The following lemma describes this equilibrium.

Lemma 1 *The equilibrium price in the capital resale market satisfies*

$$q_t = p_t. \tag{3.2}$$

Furthermore, agents' optimal choices in equilibrium are characterized by

$$\tilde{\imath}_t^{\prime m} = 0,\ \tilde{m}_t^{\prime m} = \tilde{\imath}_t + \tilde{m}_t \tag{3.3}$$

$$\tilde{\imath}_t^{\prime n} = \tilde{\imath}_t + \tilde{m}_t,\ \tilde{m}_t^{\prime n} = 0 \tag{3.4}$$

$$\tilde{\imath}_t = (1-\pi)\, \tilde{s}_t,\ \tilde{m}_t = \pi \tilde{s}_t \tag{3.5}$$

$$\tilde{s}_t = \frac{w_t}{1 + \theta^{\frac{1}{\rho}} \left\{ \pi (\frac{p_t}{p_{t+1}})^{1-\rho} + (1-\pi) R^{1-\rho} \right\}^{-\frac{1}{\rho}}} \tag{3.6}$$

It is easy to prove these results if we notice that (3.2) follows from an arbitrage argument. Since agents have the opportunity to reoptimize their portfolios after they have learned their relocation shocks, they will choose an initial portfolio so as to make the maximum profit from the change in the relative price of capital in terms of cash from p_t to q_t during the period. An interior portfolio choice will be optimal only if $q_t = p_t$. Further, the relationships (3.3) and (3.4) express the fact that after the relocation shock is realized, movers convert all their assets into cash, while non-movers convert all their assets into capital. With this, the capital resale market will be in equilibrium if

$$(1-\pi)(i_t + m_t) = i_t,$$

implying (3.5).

For future reference it is useful to note that a depositor with uninteme-diated savings obtains the lifetime expected utility level

$$\tilde{V} = \frac{w_t^{1-\rho}}{1-\rho}\left\{\theta^{\frac{1}{\rho}} + \left[\pi(\frac{p_t}{p_{t+1}})^{1-\rho} + (1-\pi)R^{1-\rho}\right]^{\frac{1}{\rho}}\right\}^{\rho} \equiv V\left(\frac{p_t}{p_{t+1}}, R; w_t\right). \tag{3.7}$$

This observation follows easily from Lemma 1.

3.2 Competitive Banking System

Next we consider depositors who have access to competitive banks ($N > 1$). In this arrangement, young agents deposit their entire savings (s_t) with a bank.[4] The bank then allocates these deposits among the economy's primary assets, money and capital. Let m_t and i_t be the bank's holdings of real balances and capital investments at t (per depositor). Banks behave competitively on the asset side of their balance sheet so that they take the returns on capital and money as given. On the liability side of their balance sheet, banks announce a vector of gross real returns on deposits for depositors who are relocated (d_t^m), and who are not relocated (d_t^n). Competitive banks choose m_t, i_t, d_t^m, and d_t^n, taking the choices of other banks as given. Competition among banks for depositors then implies that, in equilibrium, banks must choose these values to maximize the expected utility of a representative depositor. We now describe this expected utility.

[4] As in Diamond and Dybvig (1983), when competitive banks operate, all savings will be intermediated.

3.2.1 Savings Behavior and an Indirect Utility Function

Throughout we assume that banks have no control over the savings behavior of depositors. Thus depositors who use a bank offering the deposit return vector (d_t^m, d_t^n) will have old age consumption of $c_{2t}^j = d_t^j s_t$, $j = \{m, n\}$, and will choose their savings level, s_t, to maximize

$$\theta \frac{(w_t - s_t)^{1-\rho}}{1-\rho} + \frac{s_t^{1-\rho}}{1-\rho} \left[\pi (d_t^m)^{1-\rho} + (1-\pi) (d_t^n)^{1-\rho} \right].$$

The optimal savings of a young depositor is then given by

$$s_t = \frac{w_t}{1 + \theta^{\frac{1}{\rho}} \left\{ \pi (d_t^m)^{1-\rho} + (1-\pi) (d_t^n)^{1-\rho} \right\}^{-\frac{1}{\rho}}} \equiv s(d_t^m, d_t^n). \tag{3.8}$$

With this savings level, a young depositor obtains the lifetime expected utility level

$$V = \frac{w_t^{1-\rho}}{1-\rho} \left[\theta^{\frac{1}{\rho}} + \left[\pi (d_t^m)^{1-\rho} + (1-\pi) (d_t^n)^{1-\rho} \right]^{\frac{1}{\rho}} \right]^{\rho} \equiv V(d_t^m, d_t^n; w_t), \tag{3.9}$$

as a function of the return vector received, and young period income. Note that the function V is identical to the function defined in (3.7).

3.2.2 Equilibrium Bank Behavior

As already noted, competitive banks compete against each other for deposits. Hence, in equilibrium, m_t, i_t, d_t^m, and d_t^n must be chosen to maximize $V(d_t^m, d_t^n; w_t)$, subject to the following constraints. First, bank assets cannot exceed bank liabilities, so that

$$m_t + i_t \leq s_t; \quad t \geq 0. \tag{3.10}$$

Second, all payments to relocated agents must be made with currency. If b_t is the quantity of currency carried between periods, this implies that promised payments to relocated agents, $\pi d_t^m s_t$, must satisfy

$$\pi d_t^m s_t \leq (m_t - b_t)\left(\frac{p_t}{p_{t+1}}\right); \quad t \geq 0. \tag{3.11}$$

Finally, payments to non-relocated agents, $(1-\pi)d_t^n s_t$, must be financed out of income from the bank's capital investments, plus any reserves it carries between periods:

$$(1-\pi)d_t^n w_t \leq Ri_t + b_t(\frac{p_t}{p_{t+1}}); \quad t \geq 0. \tag{3.12}$$

In equilibrium banks choose m_t, i_t, b_t, d_t^m, and d_t^n to maximize $V(d_t^m, d_t^n; w_t)$, subject to (3.10) – (3.12) and non-negativity. Let $I_t \equiv Rp_{t+1}/p_t$ denote the gross nominal rate of interest, and let $\gamma_t \equiv m_t/s_t$ denote the bank's reserve-deposit ratio. That is, γ_t is the fraction of assets a competitive bank holds in the form of cash.

Lemma 2 *If* $I_t > 1$ *then* $b_t = 0$ *and*

$$\gamma_t = \left[1 + (\frac{1-\pi}{\pi})I_t^{\frac{1-\rho}{\rho}}\right]^{-1} \equiv \gamma(I_t). \tag{3.13}$$

It is easy to show that when cash is a dominated asset $(I_t > 1)$ then it is not optimal to carry reserves between periods $(b_t = 0)$. The optimal reserve deposit ratio of a competitive bank, $\gamma(I_t)$, is also straightforward to derive. Lemma 2 implies that

$$d_t^m = \gamma(I_t)\frac{\frac{p_t}{p_{t+1}}}{\pi} = \gamma(I_t)\frac{R}{\pi I_t} \tag{3.14}$$

holds, as does

$$d_t^n = d_t^m I_t^{\frac{1}{\rho}}. \tag{3.15}$$

Note that the "wedge" between the return received by relocated and non-relocated agents $(I_t^{1/\rho})$ depends on the nominal rate of interest. This is because, in order to insure depositors against relocation risk, banks must hold cash reserves. With positive nominal rates of interest, the holding of cash reserves involves an opportunity cost. The higher this opportunity cost, the less insurance depositors receive against the risk of relocation.

In the subsequent analysis, it will be useful to know some properties of the function $\gamma(I_t)$. These are stated in the following lemma, and can be easily verified using (3.13).

Lemma 3 *$\gamma(I_t)$ has the following properties:*

(i) $\lim_{I_t \to 1} \gamma(I_t) = \pi$;

(ii) $\lim_{I_t \to \infty} \gamma(I_t) = 0$;

(iii) $\frac{I_t \gamma'(I_t)}{\gamma(I_t)} = -\frac{1-\rho}{\rho} \left[1 - \gamma(I_t)\right]$

(iv) $\gamma'(I_t) < 0$.

3.3 Monopolistic Banking System

The last arrangement we consider is one where there is a monopolistic banking system ($N = 1$). Since we allow this bank considerable latitude to extract surplus from depositors, an important issue concerns the outside option of depositors. Here our assumption is that agents can either deal with the monopoly bank, or can engage in unintermediated savings and trade in post-relocation capital resale markets. However, they cannot combine the two; i.e. they cannot hold a portfolio of some unintermediated assets and some bank deposits. A monopolist bank may be able to preclude agents from directly holding the primary assets by setting a minimum deposit requirement that is equal to agents' savings. We do not want to go as far as allowing the monopolist to directly influence savings through this minimum deposit requirement, however. Instead, we envision the monopolist announcing the returns $\hat{d}_t^m$ and $\hat{d}_t^n$, to which agents respond either by choosing a savings level ($\hat{s}_t$) and depositing it with the bank or by avoiding the bank altogether, investing in the primary asset, and trading in the capital resale market.[5]

We also assume that the timing of events is such that the bank does not allow agents to withdraw deposits until after the post-relocation capital resale market closes. Thus agents who save through banks cannot do additional asset trading. This assumption was also implicitly made in our analysis of competitive banks. As Jacklin (1987) showed, when agents are allowed to trade in secondary markets after that they have learned their types, the insurance provision function of banks breaks down.

As in the case of a competitive bank, a monopoly bank allocates its deposits between cash reserves and capital investments. Let $\hat{m}_t$ denote the real value of cash reserves acquired at t, and $\hat{\imath}_t$ be the real value of capital investments made at t respectively. In addition, let $\hat{\gamma}_t = \hat{m}_t/\hat{s}_t$ be the reserve-deposit ratio of a monopoly bank. The bank's profits can be written as

$$\left[R(1-\hat{\gamma}_t) - (1-\pi)\hat{d}_t^n\right] s(\hat{d}_t^m, \hat{d}_t^n). \tag{3.16}$$

That is, total profit is the product of two terms. The first term is profit per unit deposited, which is equal to the value of the bank's return on capital investments less payments to non-relocated agents. Note that if $I_t > 1$ holds, the bank will not carry reserves between periods, an observation already incorporated in (3.16). The second term is total deposits, which is chosen by depositors in response to the returns offered by the bank. The bank takes the function s as given.

A monopoly bank maximizes profits subject to the constraint that depositors do not strictly prefer to avoid intermediation and participate directly in the post-relocation asset market. The value of depositing with a bank that

[5]This formulation of the monopolist's ability to extract surplus kept our algebra manageable while preserving the main insight about the trade-off between level of savings and composition of savings.

offers a deposit return shedule continues to be given by $V(\hat{d}_t^m, \hat{d}_t^n, w_t)$. Thus the participation constraint of a young depositor can be written as

$$V(\hat{d}_t^m, \hat{d}_t^n, w_t) \geq V(\frac{p_t}{p_{t+1}}, R, w_t). \tag{3.17}$$

In summary, the monopoly bank chooses values d_t^m, d_t^n, and $\hat{\gamma}_t$ to maximize (3.16) subject to (3.17) and non-negativity constraints. The solution to this problem is described in the following lemma.

Lemma 4 *When $I_t > 1$, the monopoly bank sets*

$$\hat{\gamma}_t = \pi \left[\frac{\pi + (1-\pi) I_t^{1-\rho}}{\pi + (1-\pi) I_t^{\frac{1-\rho}{\rho}}} \right]^{\frac{1}{1-\rho}} \equiv \hat{\gamma}(I_t) \tag{3.18}$$

and offers

$$\hat{d}_t^m = \hat{\gamma}_t (\frac{p_t}{p_{t+1}})/\pi, \tag{3.19}$$

$$\hat{d}_t^n = \hat{\gamma}_t I_t^{\frac{1}{\rho}} (\frac{p_t}{p_{t+1}})/\pi. \tag{3.20}$$

Observe that $\hat{d}_t^n = I_t^{1/\rho} \hat{d}_t^m$ continues to hold. Thus the "wedge" between the returns paid to relocated depositors and that paid to non-relocated agents is the same under monopoly versus competition. This reflects the fact that the monopolist prices efficiently, but extracts all surplus.

As in the case of the competitive bank, we summarize the properties of the reserve/deposit ratio.

Lemma 5 *$\hat{\gamma}(I_t)$ has the following properties:*

(i) $\lim_{I_t \to 1} \hat{\gamma}(I_t) = \pi$;

(ii) $\lim_{I_t \to \infty} \hat{\gamma}(I_t) = 0$;

(iii)

$$\frac{I_t\hat{\gamma}'(I_t)}{\hat{\gamma}(I_t)} = (\frac{1}{1-\rho})\frac{I_t\gamma'(I_t)}{\gamma(I_t)} + \left[\frac{(1-\pi)I_t^{1-\rho}}{\pi+(1-\pi)I_t^{1-\rho}}\right] \tag{3.21}$$

(iv) $\hat{\gamma}'(I_t) < 0.$

It remains to compare the reserve deposit ratio chosen by the monopoly bank to that chosen by a competitive bank. We have the following result.

Proposition 1 *For any* $I_t > 1$,

(i) $\hat{\gamma}(I_t) < \gamma(I_t)$,

(ii) $0 > \frac{I_t\hat{\gamma}'(I_t)}{\hat{\gamma}(I_t)} > \frac{I_t\gamma'(I_t)}{\gamma(I_t)}$.

The intuition behind the first part of the proposition is that monopoly banks do not earn profits from holding reserves, and hence a monopolistic bank holds a lower level of reserves than a competitive bank. Figure 2 illustrates the reserve deposit ratios of the two types of banking systems as a function of the nominal interest rate.

4 General Equilibrium and Economic Growth [6]

4.1 General Equilibrium with a Competitive Banking System

We start by deriving the growth rate under general equilibrium with a competitive banking system. An equilibrium with competitive banks can be de-

[6] Our interest is in comparing a competitive with a monopolistic banking system. We have introduced unintermediated investment only to describe depositors' alternatives to dealing with a monopoly bank. Therefore we do not examine a competitive equilibrium when banks do not operate, although this is easy to do.

fined as sequences of $\{k_t\}_{t=1}^{\infty}$, $\{M_t, p_t, I_t, d_t^m, d_t^n\}_{t=0}^{\infty}$ that satisfy the following conditions:

(i) The money market clears, i. e. the supply of real cash balances equals banks' demand for reserves:

$$\frac{M_t}{p_t} = \gamma(I_t)s(d_t^m, d_t^n). \tag{4.1}$$

(ii) The capital stock evolves according to

$$k_{t+1} = [1 - \gamma(I_t)]\, s(d_t^m, d_t^n). \tag{4.2}$$

(iii) The rate of return schedule (d_t^m, d_t^n) offered to depositors is given by (3.14) and (3.15).

(iv) The Fisher equation holds, i. e.

$$I_t = R\frac{p_t}{p_{t+1}}. \tag{4.3}$$

In addition, note that the nominal money stock follows the process $M_{t+1} = \sigma M_t$, the initial values of M_{-1} and k_0 are given, and the functions $\gamma(I_t)$ and $s(d_t^m, d_t^n)$ are given by (3.13) and (3.8).

In order to solve for the equilibrium of the model, our strategy will be to express all endogenous variables as a function of the nominal interest rate and then find the equilibrium sequence $\{I_t\}$. First, it will be convenient to define the savings rate of a young agent as $\eta_t \equiv s_t/w_t$. The Lemma 6 describes the properties of the savings rate.

Lemma 6 *Under a competitive banking system,*

(i) the savings rate of a young agent can be written as

$$\eta_t = \frac{1}{1 + \theta^{\frac{1}{\rho}} R^{\frac{\rho-1}{\rho}} I_t^{\frac{1-\rho}{\rho}} \left[\pi + (1-\pi) I_t^{\frac{1-\rho}{\rho}}\right]^{-1}} \equiv \eta(I_t). \tag{4.4}$$

(ii) $\eta'(I_t) < 0$ *holds.*

The first part of the lemma shows that the savings rate can simply be written as a function of the nominal interest rate. The second part implies that higher nominal interest rates reduce savings. The intuition behind this result is that higher nominal interest rates distort banks' ability to provide insurance to depositors against adverse liquidity shocks and make banks offer a less favorable real return schedule on deposits. Depositors react to this by saving less.

It follows from Lemma 6 and (2.3) that the money market clears if

$$\frac{M_t}{p_t} = \gamma(I_t)\eta(I_t)(1-\alpha)Ak_t. \tag{4.5}$$

Now we can proceed to examining the growth rate of the economy. Let us define the gross growth rate of the capital stock (and output), μ_t, as

$$\mu_t \equiv \frac{k_t + 1}{k_t}. \tag{4.6}$$

Equation (3.8) and (4.2) imply that

$$\mu_t = \eta(I_t)\left[1-\gamma(I_t)\right](1-\alpha)A \equiv \mu(I_t). \tag{4.7}$$

An important property of the function μ is stated in the following lemma.

Lemma 7 $\mu'(I_t) > 0$ *holds.*

Lemma 7 implies that higher nominal interest rates promote growth. This happens for the following reason. While higher nominal rates of interest reduce savings rates ($\eta'(I_t) < 0$ holds), they also induce banks to economize on reserves ($\gamma'(I_t) < 0$). In terms of total capital investment, the latter effect dominates. Higher nominal rates of interest induce a change in bank asset portfolio composition that increases the rate of capital accumulation. This occurs despite the decrease in the supply of deposits.

It remains to determine the equilibrium value of the nominal interest rate. Using (4.3) and (4.5) we get

$$I_t = R\frac{p_{t+1}}{p_t} = R\frac{M_{t+1}}{M_t}\frac{\gamma(I_t)\eta(I_t)k_t}{\gamma(I_{t+1})\eta(I_{t+1})k_{t+1}} = \frac{\sigma R}{\mu(I_t)}\frac{\gamma(I_t)\eta(I_t)}{\gamma(I_{t+1})\eta(I_{t+1})} \tag{4.8}$$

We focus on the balanced growth path of the economy, i. e. we look for a solution to (4.8) that satisfies $I_{t+1} = I_t = I$.[7] Along this equilibrium path we have

$$I\mu(I) = \sigma R. \tag{4.9}$$

Evidently, (4.9) gives I as an increasing function of the money growth rate, σ. The equilibrium rate of inflation is $\frac{p_{t+1}}{p_t} = \frac{I}{R}$, which is also an increasing function of σ.

Discussion

The result that higher nominal rates of interest and higher rates of inflation are conducive to real growth is in accordance with empirical evidence from economies where the rate of inflation is relatively low. Bullard and Keating (1995) and Kahn and Senhadji (2000) show that in low inflation environments, moderate increases in the rate of inflation are associated with higher long-run levels of real activity — or rates of growth. However, there is strong evidence that in environments with initially relative high rates of inflation, further increases in the rate of inflation are detrimental to real activity (Barro, 1995; Fischer, 1993; Bullard and Keating, 1995; Kahn and Senhadji, 2000). Our analysis does not permit us to address this second fact. However, it is not a difficult fact to address. By introducing an informational

[7]It is possible to verify that the only solution to (4.8) where money has positive value as $t \to \infty$ is, in fact, the balanced growth path. Paal and Smith (2002), for example, has a proof in a similar environment.

friction in credit markets that only becomes binding at high enough rates of inflation, it is easy to obtain the result that there is a "threshold" rate of inflation above which further increases in inflation have adverse consequences for real activity. Azariadis and Smith (1996) and Paal and Smith (2002) illustrate various ways in which this can be accomplished. However, we forgo this here, as our main interest is in a simple comparison of competitive versus monopolistic banking systems.

4.2 General Equilibrium with a Monopolistic Banking System

We now proceed to describe a general equilibrium when there is a monopoly bank. Equilibrium is defined as in the case of the competitive banking system, except for the following modifications. The function $\gamma(I_t)$ is replaced by $\hat{\gamma}(I_t)$, which is defined in equation (3.18), and the deposit return vector (d_t^m, d_t^n) is replaced by the vector $(\hat{d}_t^m, \hat{d}_t^n)$, which is defined in equations (3.19) and (3.20). Note that the *function* $s(.,.)$ remains the same.

Equations (3.8) and (3.18) – (3.20) imply that savings in the presence of a monopoly bank can be written as

$$\hat{s}_t = \frac{w_t}{1 + \theta^{\frac{1}{\rho}} R^{\frac{\rho-1}{\rho}} \left[\pi I_t^{\rho-1} + (1-\pi)\right]^{-\frac{1}{\rho}}} \equiv \hat{\eta}(I_t) w_t, \tag{4.10}$$

so that $\hat{\eta}(I_t)$ is the savings rate of a young depositor born at t. Note that $\hat{\eta}'(I_t) < 0$ holds, so that again higher nominal rates of interest reduce the overall savings rate.

Following the same strategy as before for constructing an equilibrium, we can now rewrite the money market clearing condition and the capital accumulation equation as

$$\frac{M_t}{p_t} = \hat{\gamma}(I_t)\hat{\eta}(I_t)(1-\alpha)Ak_t \tag{4.11}$$

and

$$\frac{k_{t+1}}{k_t} = [1 - \hat{\gamma}(I_t)]\, \hat{\eta}(I_t)(1-\alpha)A \equiv \hat{\mu}(I_t). \tag{4.12}$$

Unlike in a competitive banking system, here in general, it is not possible to unambiguously sign $\hat{\mu}'(I_t)$ for $I_t > 1$. However, Lemma 8 in the following indicates that $\hat{\mu}(I_t)$ is increasing in I_t for values of I_t that are not too large.

Lemma 8 *Under a monopolistic banking system, the growth rate of the economy satisfies*

$$\lim_{I_t \to 1} \frac{\hat{\mu}'(I_t)}{\hat{\mu}(I_t)} = \pi(\frac{1-\rho}{\rho}) \left[1 + \theta^{\frac{1}{\rho}} R^{\frac{\rho-1}{\rho}}\right]^{-1} > 0. \tag{4.13}$$

The determination of the equilibrium nominal interest rate is similar to the case with a competitive banking system. Along a balanced growth path, $I_{t+1} = I_t = I$ holds, with I satisfying

$$I\hat{\mu}(I) = \sigma R. \tag{4.14}$$

We now define the function $H(I)$ by $H(I) \equiv I\hat{\mu}(I)$. An important issue concerns circumstances under which H is an increasing function. When it is, (4.14) has a unique solution, so that a given money growth rate implies a unique nominal interest rate along the balanced growth path. [8] Lemma 9 contains a condition under which $H'(I) > 0$ holds for all $I > 1$.

Lemma 9 *Suppose that $\rho \geq \frac{\pi}{1+\pi}$ holds. Then $H'(I) > 0$ holds for any $I > 1$.*

As argued by Gomis and Smith (2003) and Schreft and Smith (2002), for the U.S., $\rho \geq \frac{\pi}{1+\pi}$ is an empirically plausible condition [9]. Thus, for the remainder of the paper, we focus on the case where H is an increasing function.

[8] It can also be shown that when $H'(I) > 0$, the unique equilibrium of the economy in which money retains positive value as $t \to \infty$ is the balanced growth path.

[9] In particular, data for the value of the reserve-deposit ratio, the interest elasticity of reserve demand, and the nominal rate of interest for the last decade imply that $\rho \approx 0.75$ and $\pi \leq 0.15$ are empirically plausible parameterizations of this economy.

4.3 Equilibrium Rates of Growth under Competition versus Monopoly

In this section we wish to compare relative rates of growth in economies with competitive versus monopolistic banking systems. In each case, we assume that policy is conducted so that the equilibrium gross nominal interest rate, I, is the same in the two economies. First we state some limiting results.

Proposition 2 *Let $\mu(I)$ be the real growth rate with a competitive banking system and $\hat{\mu}(I)$ be the real growth rate with a monopolistic banking system, as functions of the same equilibrium gross nominal interest rate, I. Then,*

(i) $\lim_{I\to 1}\hat{\mu}(I) = \lim_{I\to 1}\mu(I)$,

(ii) $\lim_{I\to 1}\hat{\mu}'(I) = \lim_{I\to 1}\mu'(I)$,

(iii) $\lim_{I\to 1}\hat{\mu}''(I) < \lim_{I\to 1}\mu''(I)$ *iff*

$$\pi + \rho < 1 \quad \textit{and} \tag{4.15}$$

$$\theta > \left(\frac{\pi\rho}{1-\pi-\rho}\right)^{\rho} R^{1-\rho}, \tag{4.16}$$

and

(iv) $\lim_{I\to\infty}\hat{\mu}(I) \leq \lim_{I\to\infty}\mu(I)$, *with stict inequality for $\theta > 0$.*

Part (i) of the proposition asserts that the real rate of growth under monopoly in banking equals the real rate of growth under competition in banking, if the Friedman rule is being followed. Parts (ii) and (iii) imply that in the neighborhood of the Friedman rule the growth rates under the two banking systems are very close (equal to a first-order approximation), and give conditions under which monopoly in banking is slightly growth-reducing (to a second-order approximation). Monopoly will hurt the growth performance of the economy — at low nominal interest rates — when there is little demand for liquidity insurance (π or ρ are low), and there is a small supply of savings (θ is low). When agents need a lot of insurance or are willing to supply ample savings, then a monopolist can extract higher profits, and, as a result, capital investments and growth will be higher. Part (iv) of the proposition states that the real growth rate under competition exceeds the real growth rate under monopoly in environments of extreme high inflation and high nominal rates of interest.

For intermediate values of the nominal interest rate, the following lemma provides an exact result for the comparison of growth rates under competition and monopoly.

Lemma 10 *$\mu(I) \geq \hat{\mu}(I)$ holds iff*

$$\begin{aligned}\pi + \theta^{\frac{1}{\rho}} R^{\frac{\rho-1}{\rho}} I^{\frac{1-\rho}{\rho}} \leq \theta^{\frac{1}{\rho}} R^{\frac{\rho-1}{\rho}} I^{\frac{1-\rho}{\rho}} & \left\{ \frac{(1-\pi) I^{\frac{1-\rho}{\rho}}}{[\pi + (1-\pi) I^{1-\rho}]^{\frac{1}{\rho}}} \right\} \\ & + \pi \left\{ \frac{\pi + (1-\pi) I^{1-\rho}}{\left[\pi + (1-\pi) I^{\frac{1-\rho}{\rho}}\right]^{\rho}} \right\}^{\frac{1}{1-\rho}} \\ & + \pi \theta^{\frac{1}{\rho}} R^{\frac{\rho-1}{\rho}} I^{\frac{1-\rho}{\rho}} \left[\frac{\pi + (1-\pi) I_t^{1-\rho}}{\pi + (1-\pi) I_t^{\frac{1-\rho}{\rho}}} \right]^{\frac{1}{1-\rho}} .\end{aligned} \tag{4.17}$$

From Lemma 10 we can see the relative growth rates under different banking systems depend on the value of nominal interest rate I, as well as the preference parameter θ, which governs savings behavior, together with other parameters. For extremely high and low propensities to save, Lemma 10 helps establish the following results.

Proposition 3 *For $1 < I < \infty$, $\hat{\mu}(I) > \mu(I)$ if θ is sufficiently small.*

Thus monopolistic banking systems lead to higher real growth rates in economies with high and relatively interest inelastic savings rates as long as the nominal interest rate is not too high. Intuitively, a monopoly bank invests a larger fraction of its assets in capital than a competitive bank. This promotes growth if the fact that monopoly banks pay lower rates of interest on deposits does not have too large an effect on the overall savings rate of depositors.

In general, it may be the case that $\hat{\mu}(I) > \mu(I)$ for some values of $I > 1$, while $\hat{\mu}(I) < \mu(I)$ for other values of $I > 1$. In other words, a monopoly in banking may lead to higher rates of real growth than would be observed with competitive banks at some, but not at other nominal rates of interest. This, and some other theoretical possibilities are illustrated via some numerical examples in Figure 3. In each panel, the value of $\mu(I)/(1-\alpha)A$ and $\hat{\mu}(I)/(1-\alpha)A$ is graphed against the nominal interest rate for various parameter combinations.

5 Welfare under Alternative Banking Arrangements

We now compare *depositor welfare* under monopolistic versus competitive banking arrangements. We assume that regardless of banking arrangements, the government maximizes a weighted sum of the ex ante expected utility of depositors, with the weight β^t assigned to the utility of the generation born at t. This leads to the following government objective functions [10]:

$$\Omega(I) \equiv \sum_{t=0}^{\infty} \beta^t V(d_t^m, d_t^n; w_t), \tag{5.1}$$

[10] Note that the government attaches no weight to the utility of the initial old generation. Paal and Smith (2002) describe a way to do so that preserves the result derived below.

in the presence of competitive banks, and

$$\hat{\Omega}(I) \equiv \sum_{t=0}^{\infty} \beta^t V(\hat{d}_t^m, \hat{d}_t^n; \hat{w}_t), \tag{5.2}$$

in the presence of a monopoly bank[11].

We now express social welfare as a function of the nominal interest rate in the following lemma [12]:

Lemma 11 **(i)** *With competitive banking,*

$$\begin{aligned}\Omega(I) = \frac{[(1-\alpha)Ak_0]^{1-\rho}}{1-\rho} \cdot \frac{[(1-\alpha)A]^{\rho}\,[1-\gamma(I)]^{\rho}\,\mu(I)^{-\rho}}{1-\beta\,[\mu(I)]^{1-\rho}} \cdot \\ R^{1-\rho} I^{\rho-1} \left[\frac{\gamma(I)}{\pi}\right]^{-\rho}.\end{aligned} \tag{5.3}$$

(ii) *With monopolistic banking,*

$$\begin{aligned}\hat{\Omega}(I) = \frac{[(1-\alpha)Ak_0]^{1-\rho}}{1-\rho} \cdot \frac{[(1-\alpha)A]^{\rho}\,[1-\hat{\gamma}(I)]^{\rho}\,\hat{\mu}(I)^{-\rho}}{1-\beta\,[\hat{\mu}(I)]^{1-\rho}} \cdot \\ R^{1-\rho} I^{\rho-1} \left[\frac{\hat{\gamma}(I)}{\pi}\right]^{1-\rho} \frac{\pi}{\gamma(I)}.\end{aligned} \tag{5.4}$$

It is now immediate from Lemma 11 that a competitive banking arrangement yields higher depositor welfare than a monopolistic banking arrangement iff

$$\begin{aligned}\left\{\frac{\mu(I)^{-\rho}}{1-\beta\,[\mu(I)]^{1-\rho}}\right\}[1-\gamma(I)]^{\rho}\,\gamma(I)^{1-\rho} > \\ \left\{\frac{\hat{\mu}(I)^{-\rho}}{1-\beta\,[\hat{\mu}(I)]^{1-\rho}}\right\}[1-\hat{\gamma}(I)]^{\rho}\,\hat{\gamma}(I)^{1-\rho}\end{aligned} \tag{5.5}$$

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[11] This social welfare function assumes that the profit of the monopoly bank is a pure loss to the society. An alternative assumption is that the government taxes part of the profit of the monopoly bank and transfer the tax revenue to depositors. We do not explore that possibility here.

[12] In order for the government's objective function to be well-defined, we must have $\beta\,[\mu(I)]^{1-\rho} < 1$, for all $I \geq 1$. This condition holds if $\beta\,[(1-\alpha)A(1-\pi)]^{1-\rho} < \left[\theta^{\frac{1}{\rho}} R^{\frac{\rho-1}{\rho}} + (1-\pi)\right]^{1-\rho}$ is satisfied. We assume throughout the paper that this condition obtains.

is satisfied. We now turn our attention to an investigation of when (5.5) does and does not hold. It turns out that the welfare ranking of competitive versus monopolistic banking systems is strongly related to the optimality or sub-optimality of the Friedman rule ($I = 1$).

The following proposition states conditions under which the Friedman rule is or is not optimal here.

Proposition 4 *When the banking system is competitive,*

(i) *Friedman rule is not optimal, i.e.,* $\lim_{I \to 1} \Omega'(I) > 0$ *holds, if* $\lim_{I \to 1} \beta \left[\mu(I)\right]^{1-\rho} > \rho$.

(ii) *Friedman rule is optimal, i.e.,* $\Omega'(I) < 0$ *holds for all* I*, if* $\lim_{I \to \infty} \beta \left[\mu(I)\right]^{1-\rho} < \rho$ *holds. This condition is equivalent to*

$$\beta \left[(1-\alpha)A(1-\pi)\right]^{1-\rho} < \rho \left[\theta^{\frac{1}{\rho}} R^{\frac{\rho-1}{\rho}} + (1-\pi)\right]^{1-\rho}. \tag{5.6}$$

Observe that the Friedman rule is sub-optimal if β or $\lim_{I \to 1} \mu(I)$ are fairly large relative to the coefficient of relative risk aversion. This kind of result arises here because departures from the Friedman rule lead to higher real rates of growth ($\mu'(I) > 0$), but they interfere with the provision of insurance by banks ($d_t^n = I_t^{\frac{1}{\rho}} d_t^m$). Thus whether or not the Friedman rule is optimal depends on the relative importance of growth versus risk-sharing. Proposition 4 partially describes the relative importance of these two considerations.

Moving back to our discussion on condition (5.5), it is useful to define the function $M(\mu)$ by

$$M(\mu) \equiv \frac{\mu^{-\rho}}{1 - \beta\mu^{1-\rho}}$$

and to define the function $Q(\gamma)$ by

$$Q(\gamma) \equiv \gamma^{1-\rho}(1-\gamma)^{\rho}$$

Then (5.5) reduces to

$$M[\mu(I)]Q[\gamma(I)] > M\left[\hat{\mu}(I)\right]Q\left[\hat{\gamma}(I)\right] \tag{5.7}$$

Lemma 12 states some useful properties of the functions $M(\cdot)$ and $Q(\cdot)$.

Lemma 12 **(i)** *$M'(\mu) \geq 0$ holds iff $\beta\mu^{1-\rho} \geq \rho$, with equality when $\beta\mu^{1-\rho} = \rho$;*

(ii) *$Q'(\gamma) \leq 0$ holds iff $\gamma \geq 1-\rho$, with equality when $\gamma = 1-\rho$.*

Lemma 12 has several immediate corollaries. The first is that, if

$$\lim_{I\to 1}\beta[\mu(I)]^{1-\rho} = \lim_{I\to 1}\beta[\hat{\mu}(I)]^{1-\rho} > \rho,$$

then $M[\mu(I)] > M[\hat{\mu}(I)]$ is equivalent to $\mu(I) > \hat{\mu}(I)$. In other words, if the Friedman rule is not optimal in a competitive banking system, the value of $M(\cdot)$ is highest for the banking system that yields the highest real growth rate. On the other hand , if

$$\lim_{I\to\infty}\beta[\mu(I)]^{1-\rho} \leq \rho$$

holds, then the Friedman rule is optimal in a competitive banking system. In that case, $M[\mu(I)] > M[\hat{\mu}(I)]$ is equivalent to $\mu(I) < \hat{\mu}(I)$. Thus, whether or not the Friedman rule is optimal in a competitive banking system (partially) determines whether the government would or would not tend to favor the banking system that leads to the highest real growth rate (for a given value of I). This should seem intuitive: if a monopoly in banking leads to higher real growth than competition in banking, then monopoly banking can only be desirable if the Friedman rule is not optimal. The higher growth rate attained under monopoly comes at the expense of less risk sharing. This can only be desirable if the Friedman rule is not optimal.

We now state some formal results about the welfare ranking for depositors of monopolistic versus competitive banking systems.

Proposition 5 **(i)** $\lim_{I \to 1} \Omega(I) = \lim_{I \to 1} \hat{\Omega}(I)$

(ii) $\lim_{I \to \infty} \Omega(I) = \lim_{I \to \infty} \hat{\Omega}(I)$

(iii) $\lim_{I \to 1} \Omega'(I) = \lim_{I \to 1} \hat{\Omega}'(I)$.

(iv) *Suppose that* $\hat{\gamma}(I) \geq 1 - \rho$. *Then if* $M'(\mu) > 0$ *and* $\hat{\mu}(I) > \mu(I)$, *or if* $M'(\mu) < 0$ *and* $\hat{\mu}(I) < \mu(I)$, *depositor welfare is higher with a monopolistic than with a competitive banking system.*

(v) *Suppose that* $\gamma(I) \leq 1 - \rho$. *Then if* $M'(\mu) > 0$ *and* $\mu(I) > \hat{\mu}(I)$, *or if* $M'(\mu) < 0$ *and* $\mu(I) < \hat{\mu}(I)$, *depositor welfare is higher under a competitive than under a monopolistic banking system.*

Proposition 5 asserts several things. Parts (i) to (iii) of the proposition indicate that whether an economy's banking system is monopolistic or competitive is not very relevant — from a welfare perspective — in environments with very low or very high nominal rates of interest. Parts (iv) and (v) of the proposition give sufficient conditions under which a monopolistic and a competitive banking system, respectively, is the preferred arrangement. These parts of the proposition are intended only to illustrate possibilities. As we show in the next section, for some specific economy, we may need to check condition (5.5) directly to compare welfare under different banking systems.

6 Illustrative Calculation of Growth and Welfare for the United States and Japan

In this section, we illustrate some of our results using the examples of two banking systems: one that can be thought of as fairly competitive – the

United States in the 1990s, and one that is highly monopolized – Japan in the 1960s. [13].

The purpose of this section is to show how our modeling framework could help to answer questions such as: what would happen to the reserve ratio and real growth rate if the United States adopted a monopolistic banking system instead of a competitive one? Would Japan have grown even faster in the 1960s if it had had a competitive banking system? From our calculations, we will see that both the United States and Japan sustain higher growth rates with their actual own banking systems. Using the results given in Section 5, we also give the conditions on certain parameter values under which competitive banking would result in higher depositor welfare for each country.

6.1 The Case of the United States

Over the 1990s, the Unites States had an average rate of inflation of about 3% per year. With a real interest rate of 4% per year (Prescott, 1986), this implies that a reasonable value for nominal interest rate I is 1.07. In addition, in order to match the aggregate reserve-to-demand-deposit ratio over the 1990s (which is 0.14, according to Gomis and Smith, 2003) and the most widely used estimate of the interest elasticity of excess reserves (-0.3; see Schreft and Smith, 2002), it is appropriate to set $\pi = 0.14$ and $\rho = 0.74$ [14]. It is then straightforward to verify that $\hat{\gamma}(1.07) = 0.139996$.

[13] Although there were more than one bank in Japan in the 1960s, it is generally believed that Japan had a highly concentrated banking system and competition among banks was very limited then (Hoshi and Kashyap, 2002). On the other hand, although national branching had not been widely adopted by U.S. banks in the 1990s, there was still a considerable degree of competition among local bank offices.

[14] These numbers are obtained under the assumption that the U.S. banking system is competitive.

Thus moving between a competitive versus a monopolistic banking system in the U.S. would mean little for the reserve-deposit ratio.

The ratio of total U.S. savings to GDP in the first quarter of 2000 was 0.182. With a value for labor's share of 0.7, and given the other values reported above, this implies that $\theta = 2.1883$ by equation (4.4). Then, if the average real growth rate of the U.S. economy was 3% per year — as it approximately was over the 1990s — this implies that $A = 6.58$. Then it is straightforward to calculate that $\hat{\mu}(1.07) = 1.029998$. This suggests that moving to a monopolized banking system in the United States — at current nominal rates of interest — would lower the real growth rate, though not by a significant magnitude.

Finally, since $1-\rho > \gamma(1.07) > \hat{\gamma}(1.07)$, we have $Q[\gamma(1.07)] > Q[\hat{\gamma}(1.07)]$. And $M(\gamma)$ is increasing in γ if $\beta\mu(1)^{1-\rho} > \rho$, according to Lemma 12. Since $\mu(1) = 1.0291$, this condition is satisfied if $\beta > .7359$. If this is true, by Proposition 5, depositor welfare in an economy like the United States is higher with a competitive than with a monopolistic banking system.

6.2 The Case of Japan

Similarly, as an example of monopolistic banking system, we examine Japan in the 1960s [15].

Over the 1960s, Japan's average annual inflation rate, measured by GDP deflator, was around 6%. We use the average of highest and lowest interbank rate as an estimate of nominal interest rate I, which is on average 1.09 per year. These imply that real interest rate is around 1.03 per year. Japan introduced reserve regulation on commercial banks since 1959 and set around

[15] Data on Japan are from various issues of Bank of Japan's "Annual Report of Economic Statistics" and "Report on National Accounts from 1955 to 1998" published by the Economic and Social Research Instititute, Cabinet Office, Government of Japan, Tokyo, 2001.

1.59% on average for banks' demand deposits over the 1960s. Most of the banks only held the minimum amount of reserves required by the Bank of Japan, so it is hard for us to estimate the interest elasticity of excess reserve holdings. For now, we just take the equilibrium reserve-deposit ratio of a Japanese bank ($\hat{\gamma}$) to be the regulated one – 1.59%, and assume Japan depositors had the same coefficient of relative risk aversion (ρ) with U.S. depositors in our previous example, which is 0.74[16]. Then, solving (3.18) for π results in $\pi = 0.0165$, much fewer early withdrawals compared with the U.S. case, which seemed to be true. Under these circumstances, if Japan had adopted a competitive banking system instead, then banks would have held slightly more reserves, with $\gamma(1.09) = 0.016$.

With average savings rate 34.6% and labor's share 0.53 for Japan in the 1960s, we can derive θ to be 0.6317. Over the 1960s, the average real GDP growth rate in Japan is around 10.4% per year. This implies that $A = 3.2423$. Then, $\mu(1.09) = 1.1038$. Japan's real GDP would have grown slower by 0.02% if its banking system had been competitive. Compared with a 0.0002% difference in the growth rates in U.S. case, this 0.02% difference is relatively larger, implying that banking structure has more significant effect on economic development in Japan than in the United States Such larger effect mainly comes from two resources: first, the saving rate is much higher in Japan than in the United States. Note that θ is calibrated to be smaller than one, meaning that in the 1960s, depositors actually valued their second period consumption more than their present consumption. This kind of saving behavior gave banks the essential role of allocating capital resources, especially given that the capital market was not well developed at the time.

[16]If the optimal choice of reserve-deposit ratio of Japanese banks were lower than the required reserve-deposit ratio, our consequent result on growth comparison would become even more significant. Moreover, because the reserve requirement of Bank of Japan was so low, we believe it was close to the equilibrium reserve holdings of Japanese banks.

Second, Japan was experiencing a higher inflation rate and also a higher nominal interest rate, than the United States. By our model's prediction, a monopoly bank has more market power in Japan than it would have in the United States, thus the difference caused by different banking structure is more significant in Japan than in the United States.

As to the welfare analysis, $\hat{\gamma}(I) < \gamma(I) < 1 - \rho$ still holds for Japan, so $Q(\hat{\gamma}) < Q(\gamma)$ by Lemma 12. Then, together with Proposition 5, if $M'(\mu) < 0$, i. e., $\beta\hat{\mu}(I)^{1-\rho} < \rho$ (equivalently, $\beta < 0.7201$), then competitive banking will result in higher welfare for Japan than monopolistic banking. However, if $\beta > 0.7201$ holds, which is likely the case, then the Friedman Rule may not be optimal and monopolistic banking may be beneficial to depositor welfare since it brings higher growth rate. By checking condition (5.5) directly, we find that if $\beta \in (0.957, 0.9745)$, monopolistic banking results in higher welfare, but not in other ranges of β.

7 Conclusion

In this paper we used a general equilibrium model to investigate how the degree of competition in deposit markets affects economic growth and depositor welfare. One of our most important results is that monopoly in banking can potentially be growth promoting. This can occur because a monopolistic bank has incentives to allocate a higher fraction of its assets to more profitable, illiquid capital investment projects than a competitive bank. However, the lower deposit interest rates offered by a monopolistic bank reduce depositors' willingness to save and thereby reduce the supply of deposits to the bank. When the first (asset allocation) effect dominates, monopoly in baking results in a higher rate of capital investment and a higher rate of growth. Our numerical illustration indicates that Japan was in such a situation in the 1960s: competition in banking would have reduced the rate of economic growth there.

Our second important result is that the degree to which a monopolistic bank can take advantage of its position depends on monetary policy. At either very low or very high nominal interest rate levels, the monopolistic bank enjoys little market power and equilibrium outcomes in the two systems are very similar. This implies that only when the Friedman rule (zero nominal interest rate) is not optimal and hyperinflation is not present, is it possible to have higher growth and higher social welfare with a monopolistic banking system.

Our model leads to several empirically testable hypotheses. First, we show that given the same rate of return from capital investment, compared with a competitive bank, a monopolistic bank always allocates a larger proportion of deposits to capital investment instead of to cash reserves. This prediction could be tested with a cross-sectional analysis of individual bank-level data. Second, we predict that the magnitude of difference between competitive and monopolistic banking systems depends on the level of the nominal interest rate as well as the interest rate elasticity of deposits. These predictions more naturally point to a cross-country analysis. Both of these empirical extensions are beyond the scale of this paper. Neverthless, we hope our theoretical findings could add some insights for further empirical studies.

Appendix

A Proof of Lemma 4

Let λ_t be a Lagrange multiplier associated with the constraint (3.17) at t. Then the bank's first order condition for its choice of $\hat{\gamma}_t$ is

$$R = \lambda_t \pi^\rho (\frac{p_t}{p_{t+1}})^{1-\rho} \hat{\gamma}_t^{-\rho} \tag{A.1}$$

The first order condition associated with d_t^n is

$$d_t^n = \lambda_t^{\frac{1}{\rho}} \tag{A.2}$$

Solving (A.1) for $\hat{\gamma}_t$, and substituting the result along with (A.2) into (3.17) yields the relation

$$\lambda_t = \Big[\frac{\pi + (1-\pi)I_t^{1-\rho}}{\pi + (1-\pi)I_t^{\frac{1-\rho}{\rho}}}\Big]^{\frac{\rho}{1-\rho}} I_t(\frac{p_t}{p_{t+1}})^{\rho} \tag{A.3}$$

Substituting (A.3) into (A.1) and solving for $\hat{\gamma}_t$ yields (3.18). And, equations (A.2) and (A.3) imply that (3.20) holds.

B Proof of Lemma 5

(i) Obvious.

(ii) L'Hospital's rule implies that

$$\begin{aligned}\lim_{I_t\to\infty} \hat{\gamma}(I_t)^{1-\rho} &= \lim_{I_t\to\infty} \pi\Big[\frac{\pi + (1-\pi)I_t^{1-\rho}}{\pi + (1-\pi)\,I_t^{\frac{1-\rho}{\rho}}}\Big] \\ &= \lim_{I_t\to\infty} \rho I_t^{-\frac{(1-\rho)^2}{\rho}} \\ &= 0\end{aligned}$$

(iii) Note that

$$\hat{\gamma}(I_t) = \pi[1 + (\frac{1-\pi}{\pi})I_t^{1-\rho}]^{\frac{1}{1-\rho}}\gamma(I_t)^{\frac{1}{1-\rho}} \tag{B.1}$$

Then, differentiating (B.1) with respect to I_t yields equation (3.21).

(iv) Equation (3.21) and Lemma 3 imply that

$$\begin{aligned}\frac{I_t\hat{\gamma}'(I_t)}{\hat{\gamma}(I_t)} &= -\frac{1}{\rho}[1-\gamma(I_t)] + \frac{(1-\pi)I_t^{1-\rho}}{\pi + (1-\pi)I_t^{1-\rho}} \\ &= -\frac{1}{\rho}\Big[\frac{(1-\pi)I_t^{\frac{1-\rho}{\rho}}}{\pi + (1-\pi)I_t^{\frac{1-\rho}{\rho}}}\Big] + \frac{(1-\pi)I_t^{1-\rho}}{\pi + (1-\pi)I_t^{1-\rho}}\end{aligned}$$

Because $\rho < 1$ and $\frac{(1-\pi)I_t^{\frac{1-\rho}{\rho}}}{\pi+(1-\pi)I_t^{\frac{1-\rho}{\rho}}} > \frac{(1-\pi)I_t^{1-\rho}}{\pi+(1-\pi)I_t^{1-\rho}}$, $\frac{I_t\hat{\gamma}'(I_t)}{\hat{\gamma}(I_t)} < 0$ holds, so does $\hat{\gamma}'(I_t) < 0$.

C Proof of Proposition 1

(i) Lemma 2 and Lemma 4 imply that $\gamma(I_t) \geq \hat{\gamma}(I_t)$ holds iff

$$[\pi + (1-\pi)I_t^{\frac{1-\rho}{\rho}}]^{\frac{\rho}{1-\rho}} - [\pi + (1-\pi)I_t^{1-\rho}]^{\frac{1}{1-\rho}} \geq 0 \tag{C.1}$$

It is easy to check that (C.1) holds with equality at $I_t = 1$. In addition, the derivative of the left-hand side of (C.1) is positive iff

$$[\pi+(1-\pi)I_t^{\frac{1-\rho}{\rho}}]^{\frac{\rho}{1-\rho}} \left[\frac{I_t^{\frac{1-\rho}{\rho}}}{\pi + (1-\pi)I_t^{\frac{1-\rho}{\rho}}}\right] \geq [\pi+(1-\pi)I_t^{1-\rho}]^{\frac{1}{1-\rho}} \left[\frac{I_t^{1-\rho}}{\pi + (1-\pi)I_t^{1-\rho}}\right] \tag{C.2}$$

Since $\frac{I_t^{\frac{1-\rho}{\rho}}}{\pi+(1-\pi)I_t^{\frac{1-\rho}{\rho}}} \geq \frac{I_t^{1-\rho}}{\pi+(1-\pi)I_t^{1-\rho}}$ holds (with strict inequality when $I_t > 1$), (C.2) holds whenever (C.1) holds. It follows that (C.1) cannot be violated for any value of I_t.

(ii) Straightforward from Lemma 3 and Lemma 5.

D Proof of Lemma 7

Combining (3.13) and (4.4), we can obtain an expression for the function μ :

$$\mu\left(I_t\right) \equiv \frac{(1-\alpha)A(1-\pi)}{\theta^{\frac{1}{\rho}}R^{\frac{\rho-1}{\rho}} + I_t^{\frac{\rho-1}{\rho}}\left[\pi + (1-\pi)I_t^{\frac{1-\rho}{\rho}}\right]} \tag{D.1}$$

Clearly, $\mu'(I_t) > 0$ holds.

E Proof of Lemma 8

Since

$$\hat{\mu}(I_t) = \frac{(1-\alpha)A[1-\hat{\gamma}(I_t)]}{1+\theta^{\frac{1}{\rho}}R^{\frac{\rho-1}{\rho}}[\pi I_t^{\rho-1}+(1-\pi)]^{-\frac{1}{\rho}}} \tag{E.1}$$

holds, it follows that (suppressing subscript t of I)

$$\begin{aligned}
\frac{I\hat{\mu}'(I)}{\hat{\mu}(I)} &= -\left[\frac{I\hat{\gamma}'(I)}{\hat{\gamma}(I)}\right]\left[\frac{\hat{\gamma}(I)}{1-\hat{\gamma}(I)}\right] - \left(\frac{1-\rho}{\rho}\right)\frac{\theta^{\frac{1}{\rho}}R^{\frac{\rho-1}{\rho}}[\pi I^{\rho-1}+(1-\pi)]^{-\frac{1}{\rho}}\left[\frac{\pi I^{\rho-1}}{\pi I^{\rho-1}+(1-\pi)}\right]}{1+\theta^{\frac{1}{\rho}}R^{\frac{\rho-1}{\rho}}[\pi I^{\rho-1}+(1-\pi)]^{-\frac{1}{\rho}}} \\
&= \left(\frac{1}{1-\rho}\right)\left(\frac{1-\rho}{\rho}\right)[1-\gamma(I)]\left[\frac{\hat{\gamma}(I)}{1-\hat{\gamma}(I)}\right] - \\
&\quad \left(\frac{1-\rho}{\rho}\right)\frac{\theta^{\frac{1}{\rho}}R^{\frac{\rho-1}{\rho}}[\pi I^{\rho-1}+(1-\pi)]^{-\frac{1}{\rho}}\left[\frac{\pi I^{\rho-1}}{\pi I^{\rho-1}+(1-\pi)}\right]}{1+\theta^{\frac{1}{\rho}}R^{\frac{\rho-1}{\rho}}[\pi I^{\rho-1}+(1-\pi)]^{-\frac{1}{\rho}}} \\
&\quad -\left[\frac{1-\pi}{\pi I^{\rho-1}+(1-\pi)}\right]\left[\frac{\hat{\gamma}(I)}{1-\hat{\gamma}(I)}\right],
\end{aligned} \tag{E.2}$$

where the second equality follows from part (iii) of Lemma 5. Now evaluate $\frac{I\hat{\mu}'(I)}{\hat{\mu}(I)}$ at $I=1$, and use $\lim_{I\to1}\gamma(I)=\lim_{I\to1}\hat{\gamma}(I)=\pi$ to get

$$\begin{aligned}
\lim_{I\to1}\frac{\mu'(I)}{\mu(I)} &= \frac{\pi}{\rho} - \pi\left[\frac{1-\rho}{\rho}\right]\left[\frac{\theta^{\frac{1}{\rho}}R^{\frac{\rho-1}{\rho}}}{1+\theta^{\frac{1}{\rho}}R^{\frac{\rho-1}{\rho}}}\right] - \pi \\
&= \pi\left[\frac{1-\rho}{\rho}\right]\left[\frac{1}{1+\theta^{\frac{1}{\rho}}R^{\frac{\rho-1}{\rho}}}\right].
\end{aligned}$$

F Proof of Lemma 9

We have

$$\begin{aligned}\frac{IH'(I)}{H(I)} &= 1+\frac{I\hat{\mu}'(I)}{\hat{\mu}(I)} \\ &= 1-\left[\frac{I\hat{\gamma}'(I)}{1-\hat{\gamma}(I)}\right] \\ &\quad -\left(\frac{1-\rho}{\rho}\right)\frac{\theta^{\frac{1}{\rho}}R^{\frac{\rho-1}{\rho}}\left[\pi I^{\rho-1}+(1-\pi)\right]^{-\frac{1}{\rho}}\left[\frac{\pi I^{\rho-1}}{\pi I^{\rho-1}+(1-\pi)}\right]}{1+\theta^{\frac{1}{\rho}}R^{\frac{\rho-1}{\rho}}\left[\pi I^{\rho-1}+(1-\pi)\right]^{-\frac{1}{\rho}}},\end{aligned} \tag{F.1}$$

where the second equality follows from (E.2). Since $\hat{\gamma}'(I)<0$ holds, we have that $H'(I)>0$ for any $I>1$ if

$$\left(\frac{1-\rho}{\rho}\right)\frac{\theta^{\frac{1}{\rho}}R^{\frac{\rho-1}{\rho}}\left[\pi I^{\rho-1}+(1-\pi)\right]^{-\frac{1}{\rho}}\left[\frac{\pi I^{\rho-1}}{\pi I^{\rho-1}+(1-\pi)}\right]}{1+\theta^{\frac{1}{\rho}}R^{\frac{\rho-1}{\rho}}\left[\pi I^{\rho-1}+(1-\pi)\right]^{-\frac{1}{\rho}}} \le 1 \tag{F.2}$$

for all $I>1$. Then, noting that $\pi > \frac{\pi I^{\rho-1}}{\pi I^{\rho-1}+(1-\pi)}$ holds for any $I>1$, a sufficient condition for (F.2) to be satisfied is that

$$\pi\left(\frac{1-\rho}{\rho}\right)\le 1$$

This condition is equivalent to $\rho \ge \frac{\pi}{1+\pi}$.

G Proof of Proposition 2

(i) From equations (D.1), (E.1), and part (i) of Lemma 5, taking $I_t=I$, it is straightforward to derive

$$\lim_{I\to 1}\hat{\mu}(I)=\lim_{I\to 1}\mu(I)=\frac{(1-\alpha)A(1-\pi)}{1+\theta^{\frac{1}{\rho}}R^{\frac{\rho-1}{\rho}}}. \tag{G.1}$$

(ii) By the chain rule, differentiating (D.1) and (E.1) gives

$$\begin{aligned}\mu'(I) &= (1-\alpha)A\left[\eta'(I)\left(1-\gamma(I)\right)-\eta(I)\gamma'(I)\right] \\ \hat{\mu}'(I) &= (1-\alpha)A\left[\hat{\eta}'(I)\left(1-\hat{\gamma}(I)\right)-\hat{\eta}(I)\hat{\gamma}'(I)\right].\end{aligned}$$

From Lemmas 3 and 5 we have

$$\lim_{I\to 1}\hat{\gamma}(I) = \lim_{I\to 1}\gamma(I) = \pi.$$

By equation (B.1), it is easy to show that

$$\lim_{I\to 1}\hat{\gamma}'(I) = \lim_{I\to 1}\gamma'(I) = \pi(1-\pi)\frac{\rho-1}{\rho}. \tag{G.2}$$

Also, it is straightforward to show that

$$\lim_{I\to 1}\hat{\eta}(I) = \lim_{I\to 1}\eta(I) = \left(1+\theta^{\frac{1}{\rho}}R^{\frac{\rho-1}{\rho}}\right)^{-1} \tag{G.3}$$

and

$$\lim_{I\to 1}\hat{\eta}'(I) = \lim_{I\to 1}\eta'(I) = \pi\left(\frac{\rho-1}{\rho}\right)\left(\frac{\theta^{\frac{1}{\rho}}R^{\frac{\rho-1}{\rho}}}{\left(1+\theta^{\frac{1}{\rho}}R^{\frac{\rho-1}{\rho}}\right)^2}\right). \tag{G.4}$$

As a result,

$$\lim_{I\to 1}\hat{\mu}'(I) = \lim_{I\to 1}\mu'(I) = (1-\alpha)A\pi(1-\pi)\frac{1-\rho}{\rho}\left(1+\theta^{\frac{1}{\rho}}R^{\frac{\rho-1}{\rho}}\right)^{-2}. \tag{G.5}$$

(iii) Algebraic manipulations can be used to establish the following:

$$\lim_{I\to 1}\hat{\mu}''(I) = C_1\hat{C}_2 \tag{G.6}$$

and

$$\lim_{I\to 1}\mu''(I) = C_1C_2, \tag{G.7}$$

where

$$C_1 = (1-\alpha)A\frac{(1-\pi)\pi R^{\frac{1}{\rho}}(1-\rho)}{\left(R^{\frac{1}{\rho}}+R\theta^{\frac{1}{\rho}}\right)^3\rho^2}, \tag{G.8}$$

$$\begin{aligned}\hat{C}_2 = -R^2\theta^{\frac{2}{\rho}}(1-\rho)(1-\pi-\rho)+\\ R^{1+\frac{1}{\rho}}\theta^{\frac{1}{\rho}}(2-2\rho+\rho^2-\pi(1-\rho^2))+\\ R^{\frac{2}{\rho}}(1-\pi(2-\rho-\rho^2)),\end{aligned} \tag{G.9}$$

$$C_2 = -R^{1+\frac{1}{\rho}}\theta^{\frac{1}{\rho}} + R^{\frac{2}{\rho}}(1 - 2\pi(1-\rho)). \tag{G.10}$$

Using these, we can get that

$$\lim_{I\to 1}\hat{\mu}''(I) - \lim_{I\to 1}\mu''(I) = C_1(1-\rho)\left(R^{\frac{1}{\rho}} + R\theta^{\frac{1}{\rho}}\right) \\ \left(R^{1/\rho}\pi\rho - R\theta^{1/\rho}(1-\pi-\rho)\right). \tag{G.11}$$

The result is straightforward from (G.11).

(iv) It is easy to derive

$$\lim_{I\to\infty}\hat{\mu}(I) = \frac{(1-\alpha)A}{1+\theta^{\frac{1}{\rho}}R^{\frac{\rho-1}{\rho}}(1-\pi)^{-\frac{1}{\rho}}} \tag{G.12}$$

and

$$\lim_{I\to\infty}\mu(I) = \frac{(1-\alpha)A}{1+\theta^{\frac{1}{\rho}}R^{\frac{\rho-1}{\rho}}(1-\pi)^{-1}}. \tag{G.13}$$

The result follows since $\rho < 1$.

H Proof of Lemma 10

In order to economize on notation, define

$$F \equiv \theta^{\frac{1}{\rho}}R^{\frac{\rho-1}{\rho}}I^{\frac{1-\rho}{\rho}}\left[\frac{(1-\pi)I^{\frac{1-\rho}{\rho}}}{\pi+(1-\pi)I^{\frac{1-\rho}{\rho}}}\right] \tag{H.1}$$

$$G \equiv \pi + \theta^{\frac{1}{\rho}}R^{\frac{\rho-1}{\rho}}I^{\frac{1-\rho}{\rho}}\left[\frac{\pi}{\pi+(1-\pi)I^{\frac{1-\rho}{\rho}}}\right] \tag{H.2}$$

and

$$x(I) \equiv \frac{\pi+(1-\pi)I^{1-\rho}}{\left[\pi+(1-\pi)I^{\frac{1-\rho}{\rho}}\right]^{\rho}} \tag{H.3}$$

Then it is straightforward to show that

$$\hat{\mu}(I) = \frac{(1-\alpha)A\left\{1 - \left[\frac{\pi}{\pi+(1-\pi)I^{\frac{1-\rho}{\rho}}}\right] x(I)^{\frac{1}{1-\rho}}\right\}}{1 + \theta^{\frac{1}{\rho}} R^{\frac{\rho-1}{\rho}} \left[\frac{I^{\frac{1-\rho}{\rho}}}{\pi+(1-\pi)I^{\frac{1-\rho}{\rho}}}\right] x(I)^{-\frac{1}{\rho}}}$$

Moreover, with some algebra it can be shown that the condition

$$\hat{\mu}(I) \leq \mu(I) = \frac{(1-\alpha)A\left\{1 - \left[\frac{\pi}{\pi+(1-\pi)I^{\frac{1-\rho}{\rho}}}\right]\right\}}{1 + \theta^{\frac{1}{\rho}} R^{\frac{\rho-1}{\rho}} \left[\frac{I^{\frac{1-\rho}{\rho}}}{\pi+(1-\pi)I^{\frac{1-\rho}{\rho}}}\right]}$$

is equivalent to

$$\pi + \theta^{\frac{1}{\rho}} R^{\frac{\rho-1}{\rho}} I^{\frac{1-\rho}{\rho}} \leq F x(I)^{-\frac{1}{\rho}} + G x(I)^{\frac{1}{1-\rho}} \tag{H.4}$$

Substituting (H.1) – (H.3) into (H.4) and rearranging terms yields equation (4.17).

I Proof of Proposition 3

If $\theta = 0$, (4.17) holds as an equality for $I = 1$, and is violated for $I > 1$. These observations follow from the fact that, with $\theta = 0$, (4.17) reduces to

$$\pi \leq \pi\, x(I)^{\frac{1}{1-\rho}},$$

where $x(I)$ as in (H.3) and $x(I) < 1$ for $I > 1$. To see this, note that first $x(1) = 1$ is obvious. In addition, differentiation yields

$$\frac{Ix'(I)}{x(I)} = (1-\rho)(1-\pi)\left\{\frac{I^{1-\rho}}{\pi + (1-\pi)I^{1-\rho}} - \left[\frac{I^{\frac{1-\rho}{\rho}}}{\pi + (1-\pi)I^{\frac{1-\rho}{\rho}}}\right]\right\}.$$

Thus, $x'(I) \leq 0$ because

$$\frac{I^{\frac{1-\rho}{\rho}}}{\pi + (1-\pi)I^{\frac{1-\rho}{\rho}}} \geq \frac{I^{1-\rho}}{\pi + (1-\pi)I^{1-\rho}},$$

with strict inequalities if $I > 1$.

The result then follows from the continuity of (4.17) in θ.

J Proof of Lemma 11

(i) Note that

$$\Omega(I)(1-\rho)[(1-\alpha)Ak_0]^{\rho-1} = \frac{\left\{1+\theta^{\frac{1}{\rho}}R^{\frac{\rho-1}{\rho}}I^{\frac{1-\rho}{\rho}}[\pi+(1-\pi)I^{\frac{1-\rho}{\rho}}]^{-1}\right\}^{\rho}}{1-\beta[\mu(I)]^{1-\rho}}.$$

$$R^{1-\rho}[(1-\pi)+\pi I^{\frac{\rho-1}{\rho}}]^{\rho}$$

$$= \left\{\frac{[(1-\alpha)A]^{\rho}[1-\gamma(I)]^{\rho}\mu(I)^{-\rho}}{1-\beta[\mu(I)]^{1-\rho}}\right\} R^{1-\rho}I^{\rho-1}\left[\frac{\pi}{\gamma(I)}\right]^{\rho} \tag{J.1}$$

Rearranging terms in (J.1) yields the expression for $\Omega(I)$ in the lemma.

(ii) It is easy to verify that the ex ante expected utility of the young generation of depositors born at t is given by

$$\frac{w_t^{1-\rho}}{1-\rho}\left\{\theta^{\frac{1}{\rho}}+\left[\pi(d_t^m)^{1-\rho}+(1-\pi)(d_t^n)^{1-\rho}\right]^{\frac{1}{\rho}}\right\}^{\rho} =$$

$$\frac{w_t^{1-\rho}}{1-\rho}\left\{\theta^{\frac{1}{\rho}}+\left[\pi(\frac{p_t}{p_{t+1}})^{1-\rho}+(1-\pi)R^{1-\rho}\right]^{\frac{1}{\rho}}\right\}^{\rho}$$

where the equality follows from the fact that (3.17) binds in the solution to the monopoly bank's problem. The government's objective function with a monopoly in banking then becomes

$$
\begin{aligned}
\hat{\Omega}(I) &= \sum_{t=0}^{\infty} \beta^t (\frac{w_t^{1-\rho}}{1-\rho}) \left\{ \theta^{\frac{1}{\rho}} + \left[\pi (\frac{p_t}{p_{t+1}})^{1-\rho} + (1-\pi) R^{1-\rho} \right]^{\frac{1}{\rho}} \right\}^{\rho} \\
&= \frac{[(1-\alpha)Ak_0]^{1-\rho}}{1-\rho} \cdot \frac{\left\{ \theta^{\frac{1}{\rho}} + \left[\pi (\frac{p_t}{p_{t+1}})^{1-\rho} + (1-\pi) R^{1-\rho} \right]^{\frac{1}{\rho}} \right\}^{\rho}}{1-\beta\left[\hat{\mu}(I)\right]^{1-\rho}} \\
&= \frac{[(1-\alpha)Ak_0]^{1-\rho}}{1-\rho} \cdot \frac{\hat{\eta}(I)^{-\rho}}{1-\beta\left[\hat{\mu}(I)\right]^{1-\rho}} \cdot R^{1-\rho} I^{\rho-1} \left[\pi + (1-\pi) I^{1-\rho} \right] \\
&= \frac{[(1-\alpha)Ak_0]^{1-\rho}}{1-\rho} \cdot \frac{[(1-\alpha)A]^{\rho} \left[1-\hat{\gamma}(I)\right]^{\rho} \hat{\mu}(I)^{-\rho}}{1-\beta\left[\hat{\mu}(I)\right]^{1-\rho}} \cdot \\
&\quad R^{1-\rho} I^{\rho-1} \left[\frac{\hat{\gamma}(I)}{\pi} \right]^{1-\rho} \frac{\pi}{\gamma(I)}.
\end{aligned}
$$

K Proof of Proposition 4

(i) Differentiating (5.3) gives

$$
\frac{I\Omega'(I)}{\Omega(I)} = (1-\rho) \left\{ \left[\frac{\beta\left[\mu(I)\right]^{1-\rho}}{1-\beta\left[\mu(I)\right]^{1-\rho}} \right] \frac{I\mu'(I)}{\mu(I)} - \frac{\pi R^{\frac{1-\rho}{\rho}} I^{\frac{\rho-1}{\rho}}}{\theta^{\frac{1}{\rho}} + (1-\pi) R^{\frac{1-\rho}{\rho}} + \pi R^{\frac{1-\rho}{\rho}} I^{\frac{\rho-1}{\rho}}} \right\} \tag{K.1}
$$

In addition, differentiating (D.1), one obtains

$$
\frac{I\mu'(I)}{\mu(I)} = \frac{(\frac{1-\rho}{\rho}) \pi R^{\frac{1-\rho}{\rho}} I^{\frac{\rho-1}{\rho}}}{\theta^{\frac{1}{\rho}} + (1-\pi) R^{\frac{1-\rho}{\rho}} + \pi R^{\frac{1-\rho}{\rho}} I^{\frac{\rho-1}{\rho}}} \tag{K.2}
$$

(K.1) and (K.2) imply that $\Omega'(1) > 0$ is satisfied iff

$$
\left[\frac{\beta\left[\mu(I)\right]^{1-\rho}}{1-\beta\left[\mu(I)\right]^{1-\rho}} \right] \left(\frac{1-\rho}{\rho} \right) > 1 \tag{K.3}
$$

holds. This is equivalent to the condition in the proposition.

(ii) Equations (K.1) and (K.2) imply that $\Omega'(I) < 0$ holds for any I iff

$$\lim_{I\to\infty} \frac{\beta\,[\mu(I)]^{1-\rho}}{1-\beta\,[\mu(I)]^{1-\rho}} < \frac{\rho}{1-\rho} \tag{K.4}$$

This condition is equivalent to the expression given in part (ii) of the proposition. Moreover,

$$\lim_{I\to\infty} \beta\,[\mu(I)]^{1-\rho} = \beta \left[\frac{(1-\alpha)A(1-\pi)}{\theta^{\frac{1}{\rho}} R^{\frac{\rho-1}{\rho}} + (1-\pi)}\right]^{1-\rho} \tag{K.5}$$

Equations (K.4) and (K.5) then imply that (5.6) must hold.

L Proof of Lemma 12

(i) Differentiating the definition of $M(\cdot)$ yields

$$\frac{\mu M'(\mu)}{M(\mu)} = \frac{(1-\rho)\beta\mu^{1-\rho}}{1-\beta\mu^{1-\rho}} - \rho$$

Part (i) of the lemma is then immediate.

(ii) Differentiating the definition of $Q(\cdot)$ yields

$$\frac{\gamma Q'(\gamma)}{Q(\gamma)} = 1 - \rho(1-\gamma)^{-1}$$

Part (ii) of the lemma then follows.

M Proof of Proposition 5

(i) This is immediate, since $\lim_{I\to 1} \mu(I) = \lim_{I\to 1} \hat{\mu}(I)$ and $\lim_{I\to 1} \gamma(I) = \lim_{I\to 1} \hat{\gamma}(I) = \pi$.

(ii) This follows from the facts that $\lim_{I\to\infty} \gamma(I) = \lim_{I\to\infty} \hat{\gamma}(I) = 0$, and $Q(0) = 0$.

(iii) This follows from part (i) of this proposition, part (ii) of Proposition 2, and (G.2).

(iv) If $\hat{\gamma}(I) \geq 1-\rho$ is satisfied, so is $\gamma(I) \geq \hat{\gamma}(I) \geq 1-\rho$. Thus $Q'[\hat{\gamma}(I)] < 0$, and $Q[\hat{\gamma}(I)] \geq Q[\gamma(I)]$ obtains.

In addition, if $M'(\mu) > 0$ holds for both $\mu(I)$ and $\hat{\mu}(I)$, and if $\hat{\mu}(I) > \mu(I)$ is satisfied, then $M[\hat{\gamma}(I)] > M[\gamma(I)]$.

On the other hand, if $M'(\mu) < 0$ holds for both $\mu(I)$ and $\hat{\mu}(I)$, and if $\hat{\mu}(I) < \mu(I)$ is satisfied, then $M[\hat{\gamma}(I)] > M[\gamma(I)]$ still holds.

In both of these cases, condition (5.7) fails.

(v) If $\gamma(I) \leq 1-\rho$, we have $Q[\gamma(I)] \geq Q[\hat{\gamma}(I)]$.In addition, if $M'(\mu) > 0$ and $\mu(I) > \hat{\mu}(I)$, or $M'(\mu) < 0$ and $\mu(I) < \hat{\mu}(I)$, then $M[\gamma(I)] > M[\hat{\gamma}(I)]$. Hence (5.7) holds.

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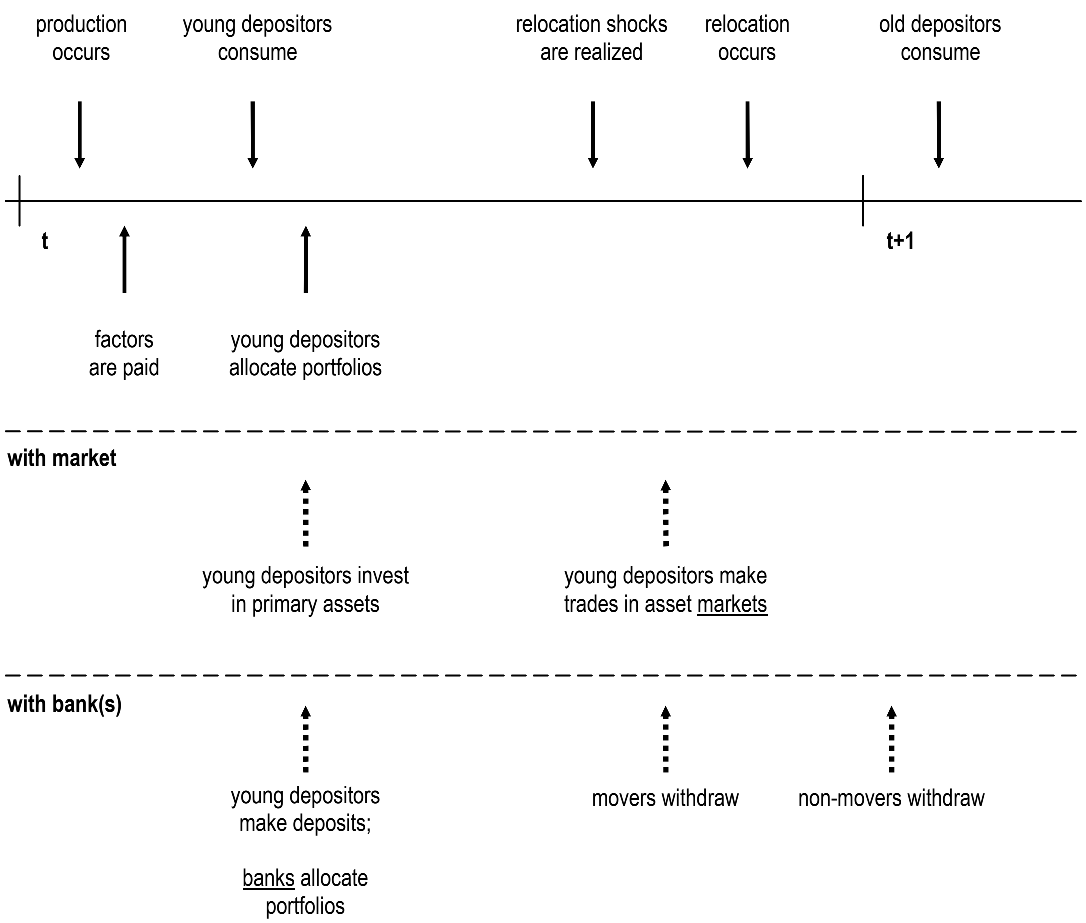

Figure 1
The timing of events

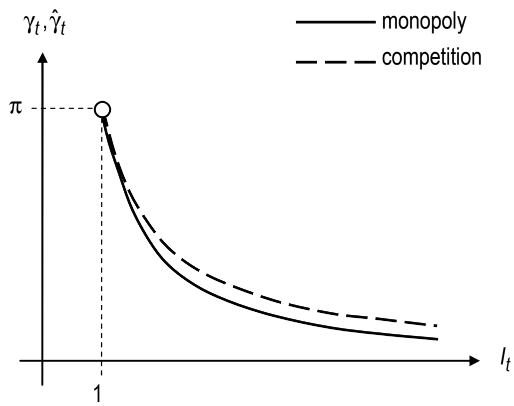

Figure 2
The reserve/deposit ratio

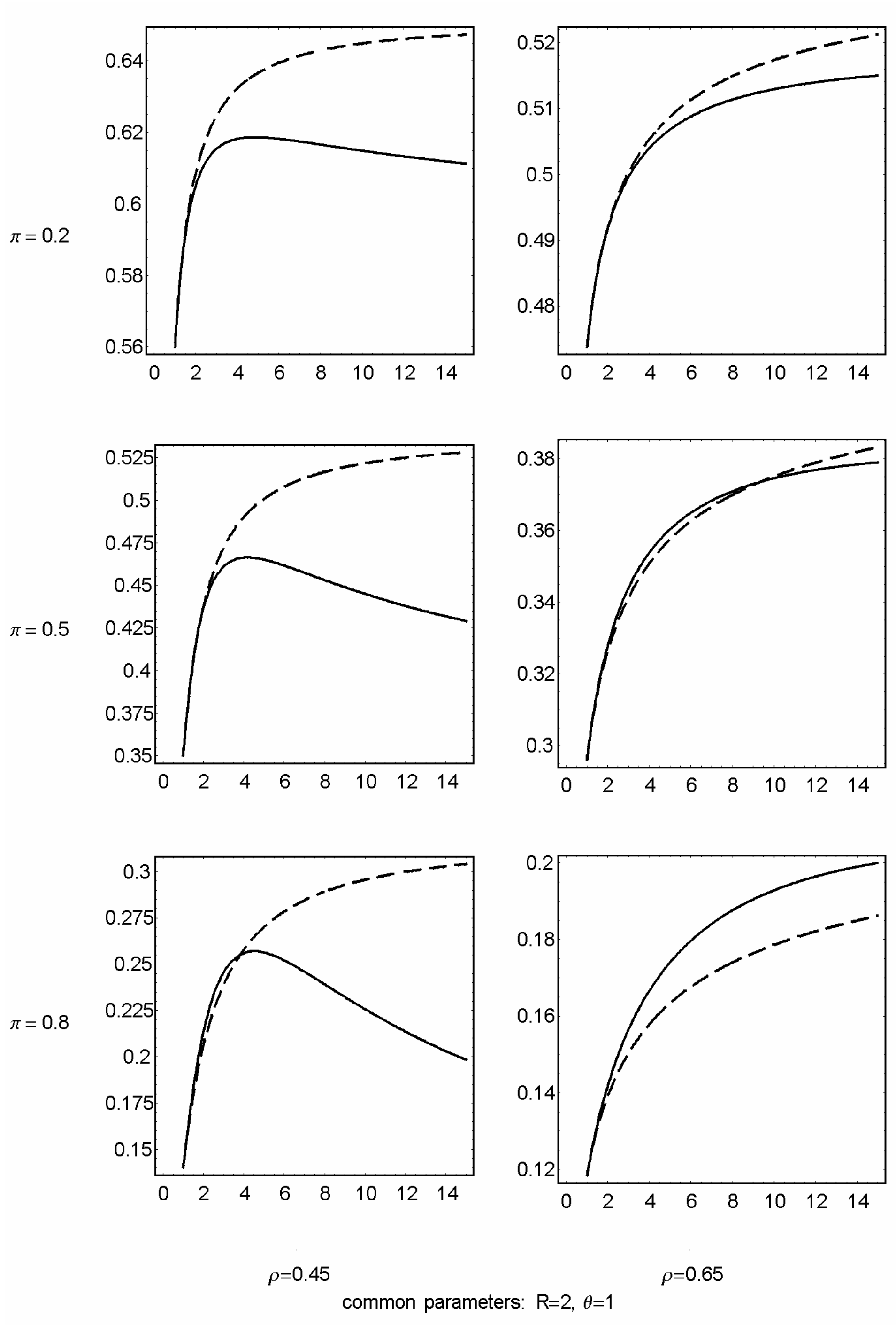

Figure 3
Growth rates as a function of the nominal interest rate
under monopoly (solid line) and competition (dashed line)