

# Growth, Inflation and the Friedman Rule

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## Abstract

According to the logic of the Friedman rule, the opportunity cost of holding money faced by private agents should equal the social cost of creating additional fiat money. Thus nominal rates of interest should be zero. This logic has been shown to be correct in a number of contexts, with and without various distortions.

In practice, however, economies that have confronted very low nominal rates of interest over extended periods have been viewed as performing very poorly rather than as performing very well. Examples include the U.S. during the Great Depression, or Japan during the last decade. Indeed economies experiencing low nominal interest rates have often suffered severe and long-lasting recessions. This observation suggests that the logic of the Friedman rule needs to be reassessed.

We consider the possibility that low nominal rates of interest imply that fiat money is a good asset. As a result, agents are induced to hold an excessive amount of savings in the form of money, and a sub-optimal amount of savings in other, more productive forms. Hence low nominal interest rates can lead to low rates of investment and, in an endogenous growth model, to low rates of real growth. This is a cost of following the Friedman rule. Benefits of following the Friedman rule include the possibility that banks will provide considerable liquidity, reducing the cost of transactions that require cash. With this trade-off, we describe conditions under which the Friedman rule is and is not optimal.

Finally, our model predicts that low rates of inflation, which are associated with low nominal rates of interest, are detrimental to growth. But it also predicts that excessively *high* rates of inflation have adverse growth consequences. This implication of the model, which is consistent with observation, in turn implies that there is a nominal rate of interest that maximizes an economy's real growth rate. We characterize this interest rate, and we describe when it is and is not optimal to drive the nominal rate of interest to its growth maximizing level.

## I. Introduction

The logic of the Friedman rule is very compelling. When nominal rates of interest are positive, individual agents perceive an opportunity cost to holding outside money. And yet, in a fiat money system, outside money is free to create from society's perspective. Hence a necessary condition for optimality is that nominal rates of interest be zero. (Friedman, 1969.)

This logic, on the other hand, need not be correct in an economy where other distortions are present. Nonetheless, the Friedman rule has been shown to be optimal in monetary economies with monopolistic competition (Ireland, 1996) and, under certain circumstances, in a variety of monetary economies where the government levies other distorting taxes (Kimbrough, 1986; Chari, Christiano, and Kehoe, 1996; Correia and Teles, 1996). In short, in a number of theoretical contexts, there seems to be a strong presumption that monetary policy should drive nominal rates of interest to zero.<sup>1</sup>

This theoretical optimality of the Friedman rule does not sit well with actual experience, however. In practice, economies that have had nominal rates of interest at or near zero have been viewed as performing quite badly, rather than performing quite well. Indeed, typical experiences with very low nominal rates of interest—like those of the U.S. during the Great Depression, or of Japan today—have been that low nominal interest rates are associated with severe and long-lasting recessions.

This disparity between theory and experience seems to call strongly for a re-evaluation of the optimality of the Friedman rule. Simple reflection suggests an obvious potential problem with that rule. When the nominal rate of interest is zero, outside money becomes a very good asset. Hence banks—or potential lenders in general—may be tempted to hold relatively large amounts of money and to make relatively few loans. If this is the case, then

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<sup>1</sup> Bryant and Wallace (1984) have demonstrated that, when a government must finance a deficit via a combination of borrowing and money creation, it may be better to issue interest-bearing debt and non-interest bearing currency than to issue only perfectly divisible, non-interest bearing currency. However, in the Bryant-Wallace set-up, full optimality requires that all agents hold only a single kind of debt issued in a minimum denomination. In other words, Bryant and Wallace do not provide a complete rationale for the co-existence of money with assets that dominate it in rate of return.

Smith (1991) and Woodford (1994) consider environments where the Friedman rule is optimal, but certain methods of implementing it lead to indeterminacies and excessive economic volatility. This line of argument suggests at least the potential for a “tension” between the determinacy and efficiency of equilibrium under a Friedman rule.

Williamson (1996) examines an economy with sequential markets and preference shocks in which the Friedman rule is not optimal. However, in his environment the presence of preference shocks, along with the sequential opening of markets, is a necessary condition for the Friedman rule not to be optimal.

low nominal rates of interest will be associated with low rates of investment, and low real rates of growth.<sup>2</sup> And, in fact, low levels of bank lending for investment purposes seem to have been a prominent feature of the Great Depression in the U.S., and are a prominent feature of the current Japanese situation.<sup>3</sup>

Our purpose in the present paper is to pursue this line of reasoning. To do so, we consider a monetary growth model with financial intermediaries. Spatial separation and limited communication create a transactions role for currency in the model, so that agents are willing to hold outside money even if it is dominated in rate of return. In addition, idiosyncratic shocks to agents' "liquidity preferences" create a role for banks to provide insurance against these shocks. In this model, the provision of insurance by banks requires them to hold cash reserves. In addition, banks make loans that fund investments in physical capital.

The optimal allocation of bank portfolios between reserves and capital depends on the nominal rate of interest. When the nominal rate is positive, banks perceive an opportunity cost of holding reserves. Under a standard assumption on preferences, the higher the nominal rate of interest, the more banks economize on reserve holdings. The more banks economize on reserve holdings, the less insurance against liquidity preference shocks they provide. This interference with insurance provision represents a distortion that arises in our model from a failure to follow the Friedman rule.

It is the case, however, that when the nominal rate of interest is low, and banks hold relatively high levels of reserves, they also fund relatively little investment in physical capital. We consider an endogenous growth model, so that low investment rates translate into low rates of real growth. This constitutes a cost of low nominal interest rates. And, as in the U.S. experience during the Depression or the Japanese experience today, low nominal interest rates are associated with low levels of bank lending to finance capital investment and with low (possibly negative) real growth rates. Notice that this is true even if low nominal rates of interest promote saving – as they do in our model. The essential issue is that low nominal interest rates cause too large a fraction of savings to be held in the form of bank reserves.

The optimal level of the nominal rate of interest in our economy is determined by trading off the benefits of bank liquidity provision (insurance) against higher rates of real growth. When the Friedman rule implies a sufficiently low real growth rate, the government will

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<sup>2</sup> See King and Levine (1993a,b) or Beck, Levine, and Loayza (2000) for evidence that bank lending to the private sector is a strong predictor of future growth performance.

<sup>3</sup> Indeed Keynes argued that money was such a good asset during the Depression that the government should actively increase the cost of holding it. Keynes proposed that this be done by means other than raising the nominal rate of interest. See, for instance, his "stamped money" proposal in "The General Theory."

not want to follow it. Instead, a benevolent government will raise the rate of money growth in order to raise nominal interest rates, and to stimulate long-run real growth.

While in our model inflation promotes growth, there is, of course, a natural limit on the extent to which money creation can be used to stimulate growth. Considerable evidence (Bullard and Keating, 1995; Khan and Senhadji, 2000) suggests that, when the rate of inflation or money creation is fairly low, modest (permanent) increases in it are conducive to higher long-run rates of real growth. However, once the long-run rate of inflation or the rate of money growth exceeds some threshold level, further increases in it actually cause growth to decline. A model that can be used to evaluate the Friedman rule, and the optimal quantity of money, should be consistent with these observations.

Our analysis enables us to state conditions under which, at low initial rates of money growth (low initial nominal interest rates), modest increases in the rate of money creation will increase the rate of real growth. When this transpires, we are also able to state conditions under which the Friedman rule is not optimal. These conditions imply that monetary policy should be used to raise the nominal rate of interest above zero as a method of stimulating growth. However, our model also has the feature that, once the rate of money creation exceeds some threshold level, further increases in it interfere with rather than promote growth. Not surprisingly, it is never optimal for the government to raise the money growth rate to the point where inflation is high enough to inhibit growth. Finally, we provide conditions under which the optimal rate of money creation maximizes the real growth rate, as well as conditions under which the optimal rate of money creation is below the growth maximizing level.

While our analysis is couched in terms of an endogenous growth model, we should emphasize that our results in no way depend on this. Analogous results can be obtained regarding steady state welfare in an economy where perpetual growth is impossible. In particular, in such an economy low nominal rates of interest continue to hinder capital investment, and therefore lead to low steady state capital stocks. The optimal steady state rate of money creation is found by trading off the benefits of higher capital stocks against the loss of liquidity provision associated with higher nominal rates of interest. The analysis that follows uses an endogenous growth formulation only because that simplifies calculations.

We should also note that throughout the analysis, we focus our attention on an economy where the government has no revenue needs. Much of the literature that is critical of the Friedman rule, beginning with Phelps (1973),<sup>4</sup> focuses on the possibility that the inflation tax is part of an optimal tax system. By considering a government that has no revenue needs, we

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<sup>4</sup> See also Kimbrough (1986), Guidotti and Vegh (1993), Correia and Teles (1996), Chari, Christiano, and Kehoe (1996), or Mulligan and Sala-I-Martin (1997).

abstract from this issue, and instead focus on the pure allocative consequences of positive nominal interest rates. Additionally, some of the literature on the sub-optimality of the Friedman rule (Levine, 1991; Wallace, 2000) has focused on the possibility that money creation is used to fund desirable programs: often programs that provide insurance against a sequence of adverse shocks. By contrast, in our model, a failure to follow the Friedman rule actually interferes with insurance provision. We think these observations make it clear that the potential sub-optimality of the Friedman rule in our environment is a pure consequence of the implications of this rule for bank portfolio allocations.

The remainder of the paper proceeds as follows. Section II describes the economic environment, while section III considers the savings behavior of young agents, the behavior of banks, and the nature of factor market transactions. Section IV discusses a full general equilibrium when nominal rates of interest are and are not positive, and section V examines when the Friedman rule is and is not optimal from a welfare perspective. Some concluding remarks are offered in section VI.

## II. The Basic Environment

### A. Production and Preferences

We consider a discrete time economy, with time indexed by  $t=0, 1, \dots$ . The economy is populated by an infinite sequence of two period lived, overlapping generations. In addition, there are two distinct locations (islands), which are described in more detail below. At each date a new generation is born on each island, consisting of a continuum of agents with unit mass.

In every period there is a single final good produced using capital and labor as factors of production. Let  $K_t$  denote the capital input, let  $L_t$  denote the labor input, and let  $k_t$  denote the capital-labor ratio of a representative producer at time  $t$ . In addition, let  $\bar{k}_t$  denote the aggregate, economy-wide average capital-labor ratio. We wish to allow for endogenous growth. To do so, we adopt the simple externalities in production formulation of Shell (1966) and Romer (1986). We therefore assume that the output ( $Y_t$ ) of a representative firm at  $t$  is given by

$$Y_t = F(K_t, L_t, \bar{k}_t) \equiv A \bar{k}_t^{1-\alpha} K_t^\alpha L_t^{1-\alpha},$$

with  $\alpha \in (0,1)$  and with  $\bar{k}_t$  taken as parametric by an individual producer.<sup>5</sup> We also assume that in the process of production capital depreciates at the rate  $\delta \in [0,1]$ .

All young agents are endowed with a single unit of labor, which they supply inelastically. They have no other endowments in any period. With respect to preferences, let  $c_{1t}$  and  $c_{2t}$  denote the first and second period consumption of a representative agent born at date  $t$ . Then we assume that this agent maximizes expected lifetime utility given by the function

$$u(c_{1t}, c_{2t}) = \frac{\theta}{1-\rho} c_{1t}^{1-\rho} + \frac{1}{1-\rho} c_{2t}^{1-\rho},$$

with  $\theta \geq 0$ , and  $\rho \in (0,1)$ .<sup>6</sup>

There are two primary assets that agents in this economy can hold. One is physical capital. One unit of the final good set aside at date  $t$  can be converted into one unit of capital at  $t+1$ ,<sup>7</sup> and, once it has been used in the production process, one unit of undepreciated capital can always be converted into one unit of consumption. The second asset is fiat money. We let the nominal per capita supply of fiat money in each location at the end of period  $t$  be denoted by  $M_t$ . This money stock grows at the exogenously selected gross rate  $\sigma$ , so that  $M_{t+1} = \sigma M_t$ . Throughout, we assume that money is injected into or withdrawn from the economy via lump-sum transfers to young agents<sup>8</sup>.

Finally, the initial old are endowed with the initial per capita capital stock,  $k_0$ , and the initial per capita money supply,  $M_{-1}$ .

## B. Transactions with Spatial Separation and Limited Communication

At the beginning of each period, every individual is assigned to one of the two locations. These locations are physically separate, and at this point there is no communication between them. As a result, trade occurs autarkically within each island.

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<sup>5</sup> We chose the simplest possible specification of technology that is consistent with sustained growth in a competitive environment. Other specifications of technology that effectively make aggregate production linear in the aggregate capital stock would deliver results similar to those reported here. Moreover, steady state analogues to our balanced growth path results can be obtained by shutting down the externality in production altogether. Thus, as it is further discussed in section V, the logic of our results does not substantially hinge on the presence of the production externality.

<sup>6</sup> The reasons for restricting  $\rho$  to be less than one, and the consequences of relaxing this restriction are described below.

<sup>7</sup> We describe the exact process of physical capital accumulation in more detail below.

<sup>8</sup> As in the models of Freeman (1985) and Abel (1987) the distribution of taxes/transfers across the different cohorts alive at a given time period affects savings behavior and capital accumulation. In this paper, we do not explore this effect, or direct intergenerational transfers in general. Instead, as we further discuss in section III, we focus on the effect of monetary policy on the operation of financial intermediaries.

The timing of events within period  $t$  is as follows. First, firms rent capital and labor and produce the final good. Final goods and undepreciated capital can either be consumed or be used to produce future capital. Young agents receive wage income, which they allocate between consumption and savings. All savings are ultimately used for the accumulation of future capital and the purchase of money balances from the old.

After production occurs, and after savings have been allocated between capital investments and cash balances, a fraction  $\pi \in (0,1)$  of young agents is “moved” to the other location. Since no inter-location communication is possible, relocated agents cannot pay for their old age consumption with checks or other privately issued claims on agents in their location of origin. This creates a role for fiat money a transactions medium, even if it is dominated by capital in rate of return. We also assume that relocation is an i.i.d. random event across agents, occurring with probability  $\pi$ . The relocation probability  $\pi$  is known at the beginning of each period, and each agent understands that he has a probability  $\pi$  of being relocated. However, the specific identities of the agents who are to be relocated are not known until after consumption has occurred and portfolios have been allocated. The stochastic nature of relocations gives rise to a demand for insurance. Our timing assumptions mean that agents cannot efficiently self-insure against these relocation shocks. This combination of circumstances implies that financial intermediaries can play a beneficial role as we now explain in more detail.

The relocation of agents occurs after goods have been consumed at  $t$ , and after capital investment occurs. Thus relocated agents have two options. One is that they can obtain currency in order to finance consumption in their new location.<sup>9</sup> The other is that they may “scrap” any capital investments, and carry the proceeds with them to their new location. We assume that one unit of capital investment scrapped at  $t$  yields  $r > 0$  units of consumption at  $t+1$ . We will want to think of  $r$  as being fairly small, so that scrapping a capital investment is a poor option. The essence of these assumptions is that relocated agents may have to liquidate higher yielding assets (claims to future capital income) in order to acquire currency or other low yielding assets. If this is the case, stochastic relocations act like “liquidity preference shocks” that force unfavorable portfolio reallocations. As in Diamond and Dybvig (1983), insurance can be efficiently provided by banks that take deposits and hold the primary assets (capital and currency reserves) of the model.<sup>10</sup>

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<sup>9</sup> This use of currency in a model of spatial separation and limited communication closely follows Townsend (1987), Mitsui and Watanabe (1989), Hornstein and Krusell (1993) and, most specifically, Champ, Smith and Williamson (1996), and Schreft and Smith (1997, 1998).

<sup>10</sup> For a comparison of alternative methods of providing insurance in this set-up, see Greenwood and Smith (1997).

The final element of our environment is the description of the capital accumulation process. We assume that the production of future capital requires the services of a young agent. Thus any resources devoted to capital investments must be allocated to young agents in the form of loans, and banks' holdings of physical capital take the form of such loans. The purpose of formulating the capital production process this way is to introduce a moral hazard problem which interferes with the operation of credit markets. Accordingly, we assume that borrowers who are relocated can simply scrap the capital investment project that they were in charge of and carry the proceeds to the new location. In this case, however, they cannot withdraw additional funds from any bank that they might have a deposit with.<sup>11</sup> By contrast, loan contracts are perfectly and costlessly enforceable if the borrower remains in the original location. Banks will structure their contracts in a way that relocating borrowers do not perceive an incentive to default on loans, and choose to withdraw cash from their deposits instead.

We summarize agents' behavior after having learned their relocation shock as follows. Agents who discover that they are to be relocated liquidate any other assets they are holding and convert the proceeds into assets that can be carried to their new location. If they wish to make cash withdrawals from a bank, first they must discharge their obligation to any bank they borrowed from by turning over control of the capital good that they produced using their loans. In contrast, agents who are not relocated are not constrained in their transactions by limitations on communication. Thus they need not withdraw from banks or scrap capital in order to consume; they can pay for consumption goods when old with checks or other credit instruments. In effect, the model provides a physical story about why some purchases are made with cash, while others are made with credit. The timing of events is summarized in Figure 1. The next section provides further details on the nature of intermediation and exchange.

### III. Transactions

#### A. Goods and Factor Markets

The exchange of capital and labor occurs within each individual location at the beginning of each period. Markets in capital and labor are competitive, implying that all factors are paid their marginal products. Thus, if  $w_t$  is the time  $t$  real wage rate and  $r_t$  is the time  $t$  capital rental rate, it follows that

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<sup>11</sup> In effect, deposits serve as collateral.

$$(1) \quad r_t = F_1(K_t, L_t, \bar{k}_t) = \alpha A; \quad t \geq 0,$$

$$(2) \quad w_t = F_2(K_t, L_t, \bar{k}_t) = (1 - \alpha) A k_t; \quad t \geq 0,$$

where the second equality in equations (1) and (2) uses the form of the production function and the equilibrium condition  $\bar{k}_t = k_t$ . Given our assumptions on capital depreciation, agents who hold capital earn the gross real return  $R_t = \alpha A + 1 - \delta \equiv R$  between periods  $t$  and  $t+1$ .

When final goods are produced, they are either consumed by the young and old agents alive at the time or are converted into capital for next period. Goods are sold in competitive markets that operate within each location at each date. The dollar price at which goods are sold at  $t$  is denoted by  $p_t$ .

## B. Government Transfers

As already noted, the government accomplishes any changes in the money stock by making lump-sum transfers to young agents. If we let  $\tau_t$  denote the transfer received by a young agent at  $t$ , then the government budget constraint requires that

$$(3) \quad \tau_t = \frac{M_t - M_{t-1}}{p_t} = \frac{\sigma - 1}{\sigma} m_t; \quad t \geq 0,$$

where  $m_t$  denotes the equilibrium stock of real money balances outstanding at  $t$ . Young agents allocate both their wage income and the transfers they receive between consumption and savings. Consequently, this method of injecting money implies that monetary policy works, in part, by affecting the supply of credit.<sup>12</sup>

## C. Banks and Savings Behavior

Young agents save some portion of their young period income ( $w_t + \tau_t$ ) and we assume that all savings are intermediated.<sup>13</sup> Intermediaries operate by announcing a deposit return schedule, denoted by  $(d_t^m, d_t)$ . Here  $d_t^m$  and  $d_t$  represent the gross real rate of interest paid on deposits between dates  $t$  and  $t+1$  to agents who are relocated and to those who are not.

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<sup>12</sup> In our view, the connection between monetary policy and credit market conditions is underemphasized in the monetary theory literature.

<sup>13</sup> In absence of the loan market friction, the assumption that all savings are deposited with banks would be innocuous. This is because, as in Diamond and Dybvig (1983), the bank would be able to implement the ex ante (unconstrained) optimal allocation, and agents would have no incentive to use other mechanisms. In the presence of the loan market friction the banking mechanism may no longer be able to achieve the unconstrained optimum. In this case, the fraction of savings deposited with banks may depend on the other options that agents have to save and insure themselves. If the other option is self-insurance (through direct asset holdings), then it is easy to show that agents still want to hold all of their savings in the form of bank deposits.

Before discussing the equilibrium determination of deposit rates of interest it is necessary to describe the savings behavior of young agents.

## 1. Savings Behavior

Suppose that a young agent at  $t$  goes to a bank offering the deposit return schedule  $(d_t^m, d_t)$ . Then, if the agent saves  $s_t$ , which he deposits with the bank, his expected utility is given by the expression

$$\frac{\theta}{1-\rho} (w_t + \tau_t - s_t)^{1-\rho} + \frac{1}{1-\rho} \left[ \pi (s_t d_t^m)^{1-\rho} + (1-\pi) (s_t d_t)^{1-\rho} \right].$$

It is useful to define the certainty equivalent gross real rate of return on bank deposits,  $\xi_t$ , as

$$(4) \quad \xi_t = \left[ \pi (d_t^m)^{1-\rho} + (1-\pi) (d_t)^{1-\rho} \right]^{\frac{1}{1-\rho}}.$$

With this, the agent's optimization problem becomes

$$\max_{s_t} \frac{\theta}{1-\rho} (w_t + \tau_t - s_t)^{1-\rho} + \frac{1}{1-\rho} (\xi_t s_t)^{1-\rho}.$$

The optimal savings behavior of a young agent at  $t$  is described by

$$(5) \quad s_t = \frac{w_t + \tau_t}{1 + \theta^{1/\rho} [\xi_t]^{(\rho-1)/\rho}} \equiv \eta_t (w_t + \tau_t),$$

where  $\eta_t$  is the savings rate. It is easy to verify that, when savings behavior is governed by (5), the expected utility of a young agent at  $t$  is

$$V(d_t^m, d_t; w_t + \tau_t) \equiv \frac{\theta}{1-\rho} (w_t + \tau_t)^{1-\rho} \left[ 1 + \theta^{-1/\rho} \xi_t^{(1-\rho)/\rho} \right]^\rho,$$

with  $\xi_t$  given by (4). Clearly, the return schedule offered by banks affects agents' behavior and welfare only through affecting the certainty equivalent rate of return. Not surprisingly,  $\partial V / \partial \xi_t > 0$ , meaning that agents will prefer arrangements that offer a higher certainty equivalent return, once their young income is fixed. We also remark that the savings rate of agents is increasing in the certainty equivalent return iff  $\rho < 1$ .

## 2. Banks

Since all savings are intermediated, all capital investments are financed through bank loans and the entire stock of money is held as bank reserves. As noted above, capital earns the gross real return  $R$  which is also the gross real rate of interest on loans. The gross real return on currency between  $t$  and  $t+1$  is  $p_t/p_{t+1}$ . Banks behave competitively in asset markets in the

sense that they take the real returns on these primary assets as given. Let  $z_t$  denote a representative bank's (per depositor) holdings of real balances, and let  $i_t$  denote the bank's (per depositor) investment in physical capital (loans). When agents deposit their time  $t$  savings,  $s_t$ , banks face the balance sheet constraint

$$(6) \quad z_t + i_t \leq s_t; \quad t \geq 0.$$

In deposit markets banks are Nash competitors: they announce gross real rates of return paid on deposits to agents who withdraw “early” (agents who move), and to agents who do not. These announcements of  $d_t^m$  and  $d_t$  are made taking the deposit returns offered by other banks, as well as the savings behavior of young agents, as given. If agents who are to be relocated are given currency to make purchases, then the bank's payments to them are constrained by its holdings of cash reserves. In particular, since the gross real return on currency is  $p_t/p_{t+1}$ , banks face the constraint

$$(7) \quad \pi \cdot d_t^m s_t \leq (z_t - b_t) \cdot \frac{p_t}{p_{t+1}}; \quad t \geq 0,$$

where  $b_t$  is the real value of cash reserves that the bank carries between  $t$  and  $t+1$ . Agents who are not relocated, on the other hand, can be paid out of any remaining income on bank assets. Consequently,

$$(8) \quad (1 - \pi) \cdot d_t s_t \leq R i_t + b_t \frac{p_t}{p_{t+1}}; \quad t \geq 0,$$

is the constraint on payments made to agents who are not relocated.

The constraints (6)–(8) are predicated on the notion that it is not optimal for the consumption of relocated agents to be funded by liquidating capital investments. A sufficient condition for this is that the return on currency exceed the return obtained by scrapping capital investments. Thus, throughout we consider equilibria with

$$(9) \quad \frac{p_t}{p_{t+1}} \geq r; \quad t \geq 0.$$

In addition, capital investments by banks must be made in the form of loans to young agents, whose efforts are required to convert current resources into future capital. We assume that these loans are made in a pro rata fashion to young agents at each date. Thus, each agent receives a loan of  $i_t$  at  $t$ . Since relocated agents have the option of scrapping capital and carrying the proceeds to their new location, these agents will repay their loans and make cash withdrawals from banks iff the payoff associated with doing so exceeds the payoff obtained by scrapping capital and consuming the proceeds at  $t+1$ . The associated incentive compatibility constraint requires that

$$(10) \quad d_t^m s_t \geq r i_t; \quad t \geq 0.$$

Equation (10) can be interpreted as a collateral constraint: the size of the loan that an agent can take out cannot exceed  $d_t^m/r$  times his deposits with the banking system. Monetary policy, through affecting the optimal choice of  $d_t^m$ , also affects the tightness of this constraint.

We assume that there is free entry into the activity of banking. In addition, we assume that all young agents simply make a deposit with a bank that offers them their most preferred deposit return schedule. Then, in a Nash equilibrium, competition among banks for depositors implies that banks will offer contracts that maximize depositor welfare, or, equivalently, maximize the certainty equivalent return on deposits. Thus the bank's problem is to choose the values  $d_t^m$ ,  $d_t$ ,  $z_t$ ,  $b_t$  and  $i_t$  in order to maximize (4) subject to the constraints (6)–(8), (10),  $z_t \geq 0$ ,  $i_t \geq 0$ , and  $b_t \geq 0$ .<sup>14</sup>

The nature of the solution to the bank's problem depends on whether or not the incentive constraint (10) is binding, and on whether or not banks perceive a positive opportunity cost of holding reserves. In particular, define the gross nominal rate of interest  $I_t$  in the conventional way;  $I_t \equiv R(p_{t+1}/p_t)$ .<sup>15</sup> Then the solution to the bank's problem can differ, depending on whether  $I_t > 1$  or  $I_t = 1$  holds. We now consider each case in turn.

### 2.1 *Bank Behavior with Positive Nominal Interest Rates ( $I_t > 1$ ) and a Non-binding Incentive Constraint*

When nominal rates of interest are positive, banks perceive an opportunity cost of carrying reserves between periods. Since their withdrawal demand is perfectly predictable,<sup>16</sup> they will never choose to do so; that is,  $b_t = 0$  will hold. In addition, if we define  $\gamma_t \equiv z_t/s_t$  to be the reserve-deposit ratio of a representative bank, it is easy to verify that the optimal choice of  $\gamma_t$  satisfies

$$(11) \quad \gamma_t = \left[ 1 + \frac{1-\pi}{\pi} I_t^{\left(\frac{1-\rho}{\rho}\right)} \right]^{-1} \equiv \gamma(I_t),$$

as long as this choice does not violate (10). It follows, then, that  $i_t = (1 - \gamma(I_t))s_t$ . For future reference, it will be useful to note that  $\gamma(1) = \pi$ . In addition, the assumption that  $\rho \in (0, 1)$  implies

<sup>14</sup> Identical results would be obtained if the intermediary was regarded as a coalition of young agents formed at each date.

<sup>15</sup>  $I_t$  is the gross nominal interest rate on loans. Analogously,  $d_t^m p_{t+1}/p_t$  and  $d_t p_{t+1}/p_t$  are the gross nominal interest rates on deposits that are withdrawn early and on those that are held to maturity. In the equilibrium of our model, it will be the case that  $d_t p_{t+1}/p_t \geq I_t \geq 1 \geq d_t^m p_{t+1}/p_t$ , at least when  $\rho < 1$ . Thus our deposits are similar to time deposits that have a nominal penalty for early withdrawal.

<sup>16</sup> See Champ, Smith, and Williamson (1996) for an analysis of stochastic withdrawal demand in this context.

that  $\gamma(I) < 0$  is satisfied. Thus the fraction of bank assets held as currency reserves is decreasing, while the fraction held as loans is increasing as a function of the nominal interest rate.<sup>17</sup> In other words, conducting monetary policy in a way that increases the equilibrium nominal interest rate will alter the composition of savings to favor capital accumulation.

The optimal values  $d_t^m$  and  $d_t$  are given by (7) and (8) at equality (and with  $b_t=0$ ), as

$$(12) \quad d_t^m = \frac{R\gamma(I_t)}{\pi I_t}$$

and

$$(13) \quad d_t = \frac{R(1-\gamma(I_t))}{1-\pi} \equiv (I_t)^{\frac{1}{\rho}} d_t^m,$$

where the second equality follows from the definition of  $\gamma(I)$ . It is easy to verify that positive nominal interest rates render it sub-optimal for banks to provide complete insurance against the event of a relocation. In other words, when  $I_t > 1$ ,  $d_t^m < d_t$  will obtain. Moreover, the higher the nominal interest rate, the larger the gap between  $d_t$  and  $d_t^m$ .

The absence of complete insurance against the event of a relocation is a consequence of the fact that relocated agents must be given currency, and that positive nominal interest rates cause agents to regard holding currency as involving an opportunity cost. The resulting failure of agents to be fully insured against the event of a relocation is a distortion induced by a failure to follow the Friedman rule. The degree of this distortion is conveniently measured by the certainty equivalent return on deposits. When the incentive constraint does not bind, (12) and (13) can be substituted into (4) to express  $\xi_t$  as

$$\xi_t = R \left[ \pi (I_t)^{\frac{\rho-1}{\rho}} + 1 - \pi \right]^{\frac{\rho}{1-\rho}} \equiv \xi(I_t)$$

It is straightforward to verify that

$$\frac{I \xi'(I)}{\xi(I)} = -\gamma(I) < 0.$$

The farther monetary policy deviates from the Friedman rule, the lower the certainty equivalent return that depositors receive. The effects of  $I_t$  on  $\xi_t$ ,  $d_t$  and  $d_t^m$  are illustrated in Figure 2.

It remains to state conditions under which the incentive constraint does not bind in the bank's problem. Since  $b_t=0$  and (8) holds with equality, (10) can be rewritten as

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<sup>17</sup> If  $\rho \geq 1$  holds, then the income effects associated with a change in the nominal interest rate dominate substitution effects, and the bank's optimal reserve-deposit ratio will be an increasing function of the rate of interest. We comment below on how this would affect our analysis. However, clearly  $\gamma < 0$  is the intuitively more appealing case.

$$(14) \quad \frac{d_t}{d_t^m} \leq \frac{r(1-\pi)}{R}.$$

Stated in this form, the incentive constraint says that non-movers cannot be promised too much consumption relative to movers. Otherwise, the benefit from scrapping the large investment projects (implied by the large funding requirements of non-movers) relative to the small return promised to movers if they were to adhere to their contracts would induce movers not to repay their loans.

The unconstrained optimal contract offered by banks, described by (12) and (13), satisfies the incentive constraint (14) iff

$$(15) \quad I_t \leq \left[ \frac{R}{(1-\pi)r} \right]^\rho \equiv \hat{I}.$$

Note that  $\hat{I} > 1$ , so that for low values of the nominal interest rate the incentive constraint is never binding. In addition, it is easy to see that equation (9) has the equivalent representation  $I_t < R/r$ . We will typically make the assumption that  $\hat{I} < R/r$ , which is equivalent to

$$(16) \quad (r/R)^{1-\rho} < (1-\pi)^\rho.$$

When this condition holds, the constraint (10) is not binding for  $I_t \in [1, \hat{I}]$ , and it is binding for  $I_t \in (\hat{I}, R/r]$ <sup>18</sup>

It will also be useful for future reference to describe the savings behavior, and the expected utility of young agents when banks behave optimally, and when the incentive constraint (10) does not bind. Equation (5) implies that the savings rate of young agents satisfies

$$\eta_t = \frac{1}{1 + \theta^{1/\rho} \xi(I_t)^{(\rho-1)/\rho}} \equiv \eta(I_t),$$

It is easy to verify that

$$\frac{I\eta'(I)}{\eta(I)} = -\frac{1-\rho}{\rho} \gamma(I) \cdot (1-\eta(I))$$

holds. When  $\rho < 1$ , low nominal rates of interest promote savings. Intuitively, this occurs because low nominal rates of interest increase the certainty equivalent return on deposits, and the savings rate is an increasing function of this return when  $\rho < 1$ . Therefore, following the Friedman rule maximizes the savings of young agents.<sup>19</sup>

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<sup>18</sup> Note that for  $\rho > 1$  (16) cannot hold, so that the incentive constraint is never binding.

<sup>19</sup> When  $\rho > 1$ , low nominal interest rates still increase the certainty equivalent return on deposits, but the savings rate is a decreasing function of this return. Thus  $\eta'(I) > 0$ .

Using what we know about the equilibrium values of  $d_t^m$  and  $d_t$ , along with the definitions of  $\xi$  and  $\eta$ , it is easy to show that the maximized value of a young agent's expected utility at  $t$  is given by

$$V\left(\frac{R\gamma(I_t)}{\pi I_t}, \frac{R(1-\gamma(I_t))}{1-\pi}; w_t + \tau_t\right) = \frac{\theta}{1-\rho} (w_t + \tau_t)^{1-\rho} \left[1 + \theta^{-1/\rho} (\xi(I_t))^{(1-\rho)/\rho}\right]^\rho \equiv \frac{\theta}{1-\rho} (w_t + \tau_t)^{1-\rho} (1-\eta(I_t))^{-\rho}; \quad 1 < I_t \leq \hat{I}.$$

Clearly, the indirect utility of a young agent with given income  $(w_t + \tau_t)$  is decreasing in the nominal interest rate. This is due to the distortionary effect of deviating from the Friedman rule, and works through a reduction in the certainty equivalent return that savers receive.

## 2.2 *Bank Behavior with Positive Nominal Interest Rates ( $I_t > 1$ ) and a Binding Incentive Constraint*

We now describe what happens when (15) is not satisfied. As before, let  $\gamma_t \equiv z_t/s_t$  be the bank's reserve-deposit ratio. Since (7) and (8) must hold with equality and  $b_t=0$  in equilibrium, it is easy to verify that the incentive constraint (10) reduces to

$$(17) \quad \gamma_t (p_t/p_{t+1})/\pi \geq r(1-\gamma_t).$$

When  $I_t \geq \hat{I}$ , (17) holds as an equality. It is then straightforward to show that the bank's optimal reserve-deposit ratio is given by

$$(18) \quad \gamma_t = \left[1 + \frac{R}{\pi r I_t}\right]^{-1} \equiv \tilde{\gamma}(I_t).$$

Note that  $\tilde{\gamma}'(I_t) > 0$  holds. Thus, in contrast to the unconstrained case, a bank that faces a binding incentive constraint responds to higher nominal interest rates by increasing rather than reducing its reserve deposit ratio.

When the incentive compatibility condition (10) binds in the banks' problem, banks must ration credit in order to prevent agents from defaulting on loans. This rationing of credit will be required whenever the nominal rate of interest is sufficiently high, signaling a high rate of inflation. Under these circumstances the return on currency is so low that relocated agents, if they received an unrestricted credit allocation, would prefer scrapping capital and defaulting on loans instead of withdrawing currency and carrying it with them to their new location. In order to overcome this moral hazard problem, banks restrict the size of the loans that they make. By doing so, they prevent agents who default (scrap capital) from obtaining excessively high levels of future consumption. Moreover, the higher the nominal rate of interest (the rate of inflation) the more severely banks will have to ration credit. As a result, as the nominal rate of interest

increases, banks make fewer rather than more loans (hold more rather than fewer reserves). Thus, for  $I_t > \hat{I}$ ,  $\tilde{\gamma}'(I_t) > 0$ .<sup>20</sup>

When the bank's reserve-deposit ratio satisfies (18), the rates of return offered to movers and non-movers are given by  $d_t^m = r(1 - \tilde{\gamma}(I_t))$ , and  $d_t = R(1 - \tilde{\gamma}(I_t))/(1 - \pi) \equiv d_t^m R/(1 - \pi)r$ . Then, for  $I_t \geq \hat{I}$ , the certainty equivalent return on deposits satisfies

$$\xi_t = \frac{\left[ \pi r^{1-\rho} + (1-\pi)^\rho R^{1-\rho} \right]^{\frac{1}{1-\rho}}}{1 + \frac{\pi r I_t}{R}} \equiv \tilde{\xi}(I_t).$$

As before, the function  $\tilde{\xi}$  describes how well banks insure agents against relocation risk when the incentive constraint (10) is binding. Since

$$\frac{I\tilde{\xi}'(I)}{\tilde{\xi}(I)} = -\tilde{\gamma}(I) < 0,$$

the loss suffered by savers in terms of certainty equivalent return continues to be increasing in the nominal interest rate. Furthermore, since  $\tilde{\gamma}(I) > \gamma(I)$ , this marginal loss is greater than what it would be if the credit market friction was not binding at that nominal interest rate. Figure 2 illustrates these points.

Repeating the same sequence of steps as previously, it is easy to check that the savings behavior of a young agent at  $t$  obeys

$$\eta_t = \frac{1}{1 + \theta^{1/\rho} \tilde{\xi}(I_t)^{(\rho-1)/\rho}} \equiv \tilde{\eta}(I_t),$$

with

$$\frac{I\tilde{\eta}'(I)}{\tilde{\eta}(I)} = -\frac{1-\rho}{\rho} \tilde{\gamma}(I) (1 - \tilde{\eta}(I)).$$

As before, the effects of the nominal interest rate on the savings rate are transmitted through the certainty equivalent return. A higher nominal interest rate reduces this return, thereby lowering the savings rate.

Finally, when the incentive constraint binds in the bank's problem, the maximized expected utility of a young agent at  $t$  is

$$V\left(\frac{R\tilde{\gamma}(I_t)}{\pi I_t}, \frac{R(1 - \tilde{\gamma}(I_t))}{1 - \pi}; w_t + \tau_t\right) = \frac{\theta}{1-\rho} (w_t + \tau_t)^{1-\rho} \left(1 + \theta^{-1/\rho} (\tilde{\xi}(I_t))^{(1-\rho)/\rho}\right)^\rho \equiv$$

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<sup>20</sup> The notion that higher rates of inflation may make credit rationing more severe is also pursued by Azariadis and Smith (1996) and Boyd and Smith (1998).

$$\frac{\theta}{1-\rho}(w_t + \tau_t)^{1-\rho}(1-\tilde{\eta}(I_t))^{-\rho}.$$

For future reference, define the function  $\hat{\gamma}$  by

$$\hat{\gamma}(I_t) \equiv \max\{\gamma(I_t), \tilde{\gamma}(I_t)\} \equiv \begin{cases} \gamma(I_t) & \text{for } 1 \leq I_t \leq \hat{I} \\ \tilde{\gamma}(I_t) & \text{for } \hat{I} < I_t \leq \frac{R}{r} \end{cases}.$$

Then the optimal reserve-deposit ratio of a representative bank is given by  $\hat{\gamma}(I_t)$  and is illustrated in Figure 3. Analogously, we define the functions  $\hat{\xi}$  and  $\hat{\eta}$  by

$$\hat{\xi}(I_t) \equiv \begin{cases} \xi(I_t); & I_t \leq \hat{I} \\ \tilde{\xi}(I_t); & I_t > \hat{I} \end{cases}$$

and

$$\hat{\eta}(I_t) \equiv \begin{cases} \eta(I_t); & I_t \leq \hat{I} \\ \tilde{\eta}(I_t); & I_t > \hat{I} \end{cases}.$$

These functions describe the equilibrium allocation of relocation risk, and the equilibrium savings rate — taking full account of whether or not the incentive constraint is binding — when nominal rates of interest are positive. The function  $\hat{\xi}(I_t)$  is illustrated in Figure 2 and the function  $\hat{\eta}(I_t)$  is illustrated in Figure 3.

### 2.3 Bank Behavior with Zero Nominal Interest Rates ( $I_t = 1$ )

When the nominal rate of interest is zero, there is no opportunity cost to carrying reserves between periods. Hence  $b_t > 0$  can hold. Moreover, it is easy to verify that each bank opts to provide complete insurance, so that  $d_t^m = d_t = R = p_t/p_{t+1}$ . In addition, banks are indifferent regarding their portfolio composition so long as their reserves are adequate to provide the insurance desired. This will be the case iff  $z_t/s_t \geq \pi$  holds. Thus, zero nominal interest rates are consistent with optimal bank behavior if and only if the per capita supply of real balances is sufficiently large.

When the nominal rate of interest is zero, it is apparent that the savings rate of young agents is  $\eta(1)$ , and that the expected utility of a young agent at  $t$  is

$$V(R, R; w_t + \tau_t) = \frac{\theta}{1-\rho}(w_t + \tau_t)^{1-\rho} \left[ 1 + \theta^{-1/\rho} R^{(1-\rho)/\rho} \right]^\rho.$$

## IV. General Equilibrium

As in the case of optimal bank behavior, the conditions of equilibrium differ depending on whether or not the nominal rate of interest is positive. Of course, it also matters whether or not banks face a binding incentive constraint with respect to loan repayments. We now describe each case in turn. We first focus on stationary equilibria – equilibria in which the nominal interest rate is constant over time, implying that the economy follows a balanced growth path. We discuss dynamical equilibria at the end of this section.

### A. Positive Nominal Interest Rates ( $I_t > 1$ ).

When banks have a determinate optimal portfolio, there are several conditions that an equilibrium must satisfy. First, the supply of and the demand for real balances must be equal, so that

$$(19) \quad m_t = \hat{\gamma}(I_t)\hat{\eta}(I_t)(w_t + \tau_t); \quad t \geq 0.$$

Second, the time  $t+1$  capital stock must equal the level of investment at date  $t$ . From the bank balance sheet constraint (6), this requires that

$$k_{t+1} = (1 - \hat{\gamma}(I_t))\hat{\eta}(I_t)(w_t + \tau_t); \quad t \geq 0.$$

Finally, the government budget constraint (3) must hold. These three conditions, together with the given initial values of  $k_0$  and  $M_{-1}$  describe the equilibrium path of the economy for a fixed money growth rate  $\sigma$  (assuming that  $I_t > 1$  holds at all times).

Conditions (19) and (3) together imply that the equilibrium level of transfers satisfies

$$\tau_t = \left[ \frac{\sigma}{(\sigma - 1)\hat{\gamma}(I_t)\hat{\eta}(I_t)} - 1 \right]^{-1} w_t$$

Combining this with (2) allows us to express the income of young agents as

$$w_t + \tau_t = \left[ 1 - \frac{\sigma - 1}{\sigma} \hat{\gamma}(I_t)\hat{\eta}(I_t) \right]^{-1} (1 - \alpha)Ak_t.$$

Therefore, the equilibrium sequences  $\{k_t\}$ ,  $\{m_t\}$ , and  $\{I_t\}$  must satisfy

$$(20) \quad m_t = \frac{\hat{\gamma}(I_t)\hat{\eta}(I_t)}{1 - \frac{\sigma - 1}{\sigma} \hat{\gamma}(I_t)\hat{\eta}(I_t)} (1 - \alpha)Ak_t; \quad t \geq 0,$$

$$(21) \quad k_{t+1} = \frac{(1 - \hat{\gamma}(I_t))\hat{\eta}(I_t)}{1 - \frac{\sigma - 1}{\sigma} \hat{\gamma}(I_t)\hat{\eta}(I_t)} (1 - \alpha)Ak_t; \quad t \geq 0,$$

and,

$$(22) \quad I_t = R \frac{p_{t+1}}{p_t} = \sigma R \frac{m_t}{m_{t+1}}; \quad t \geq 0.$$

We first describe equilibria in which  $I_t$  is constant over time. Then (21) implies that  $k_t$  is growing at a constant rate, and (20) implies that  $m_t$  is growing at the same rate. This growth rate is given by

$$(23) \quad \frac{k_{t+1}}{k_t} = \frac{m_{t+1}}{m_t} = \frac{(1 - \hat{\gamma}(I))\hat{\eta}(I)(1 - \alpha)A}{1 - \frac{\sigma-1}{\sigma}\hat{\gamma}(I)\hat{\eta}(I)}$$

Using (23), (22) can be rewritten along the balanced growth path as

$$(24) \quad I = \sigma R \frac{1 - \frac{\sigma-1}{\sigma}\hat{\gamma}(I)\hat{\eta}(I)}{(1 - \hat{\gamma}(I))\hat{\eta}(I)(1 - \alpha)A}.$$

Equation (24), if it can be solved for  $I$ , implicitly determines the nominal interest rate implied by a given money growth rate. With this, (23) determines the growth rate of the capital/labor ratio, and recursively the entire  $k_t$  sequence. Then (20) can be used to derive the sequence of real balances. Finally, the price level at  $t$  is determined by  $p_t = m_t/M_t = m_t/\sigma^{t+1}M_{-1}$ .

From a technical perspective, it is simpler to investigate the properties of (24) if we think of this equation as determining  $\sigma$  as a function of  $I$ , rather than  $I$  as a function of  $\sigma$ . Correspondingly, we can think of monetary policy in the stationary equilibrium as targeting a certain nominal interest rate, which is supported by a certain money growth rate. To derive this required money growth rate, (24) can be solved explicitly for  $\sigma$  as

$$(25) \quad \sigma = \frac{\frac{I(1-\alpha)A(1-\hat{\gamma}(I))}{R\hat{\gamma}(I)} - 1}{\frac{1}{\hat{\gamma}(I)\hat{\eta}(I)} - 1} \equiv \hat{\sigma}(I).$$

Of course, the function  $\hat{\sigma}(I)$  consists of two parts. For low target values of  $I$ , the incentive constraint is not binding and  $\hat{\sigma}(I)$  reduces to

$$(26) \quad \hat{\sigma}(I) = \frac{\eta(I)\gamma(I)}{1 - \eta(I)\gamma(I)} \left[ \frac{(1 - \pi)(1 - \alpha)A}{\pi R} I^{1/\rho} - 1 \right] \equiv \sigma(I), \quad \text{for } I \in [1, \hat{I}].$$

On the other hand, for high target values of  $I$ , the incentive constraint is binding and  $\hat{\sigma}(I)$  can be written as

$$(27) \quad \hat{\sigma}(I) = \frac{\tilde{\gamma}(I)\tilde{\eta}(I)}{1 - \tilde{\gamma}(I)\tilde{\eta}(I)} \left[ \frac{(1 - \alpha)A}{\pi R} - 1 \right] \equiv \tilde{\sigma}(I), \quad \text{for } I \in (\hat{I}, R/r].$$

The following lemma characterizes some properties of the function  $\hat{\sigma}(I)$ .

Lemma 1.

Assume that  $\rho < 1$  and  $\hat{I} < R/r$ . Then

- (a)  $\hat{\sigma}(1) = \sigma(1) = \frac{\eta(1)((1-\pi)(1-\alpha)A - \pi R)}{R(1-\pi\eta(1))}$ , and
- (b)  $\hat{\sigma}'(I) > 0$  holds for all  $I$ . Furthermore,
- (c) for  $I \leq \hat{I}$ ,  $I\sigma'(I)/\sigma(I) > 1$  holds, and  
for  $I > \hat{I}$ ,  $I\tilde{\sigma}'(I)/\tilde{\sigma}(I) \leq 1$  holds with the inequality being strict iff  $\theta > 0$ .

The proof of Lemma 1 appears in Appendix A. Part (a) of the lemma implies that there is a rate of money creation consistent with  $I=1$  iff  $(1-\pi)(1-\alpha)A > \pi R$  is satisfied. If this inequality is violated, then the money supply cannot be contracted rapidly enough to maintain a zero nominal interest rate, and, indeed, the nominal rate of interest cannot fall below the value  $[\pi R / ((1-\pi)(1-\alpha)A)]^p > 1$ . We will typically assume that  $\sigma(1) > 0$  is satisfied, so that it is feasible — if not necessarily optimal — to follow the Friedman rule. From part (b) of the lemma it follows that the function  $\hat{\sigma}(I)$  has an inverse. Therefore, along the balanced growth path, there is a one-to-one mapping between a policy of targeting a constant nominal interest rate and a policy of choosing a constant money growth rate.

Finally, part (c) of Lemma 1 asserts that increases in the rate of money creation induce a less than proportional increase in the gross nominal rate of interest when  $\sigma < \hat{\sigma}(\hat{I})$  and a more than proportional increase in the gross nominal rate of interest when  $\sigma \geq \hat{\sigma}(\hat{I})$ . This fact has important implications for how increases in the rate of money creation affect the real rate of growth, which we now turn to.

Combining (25) and (22) allows us to express the growth rate of the economy as a function of the nominal interest rate the following way:

$$(28) \quad \frac{k_{t+1}}{k_t} = \frac{m_{t+1}}{m_t} = \frac{\hat{\sigma}(I)R}{I} = \frac{\frac{(1-\alpha)A(1-\hat{\gamma}(I))}{\hat{\gamma}(I)} - 1}{\frac{1}{\hat{\gamma}(I)\hat{\eta}(I)} - 1} \equiv \hat{\mu}(I).$$

Thus the function  $\hat{\mu}(I)$  gives the equilibrium gross real rate of growth — along a balanced growth path — for a given value of the target nominal interest rate  $I$ . A simple consequence of (28) is that

$$\frac{I\hat{\mu}'(I)}{\hat{\mu}(I)} = \frac{I\hat{\sigma}'(I)}{\hat{\sigma}(I)} - 1$$

Based on this identity and part (c) of Lemma 1 we can state the following result regarding the growth rate of the economy.

Proposition 1.

Assume that  $\rho < 1$  and that the economy is along a balanced growth path with  $I > 1$ .

When  $1 < I < \hat{I}$ , then the equilibrium rate of growth is a strictly increasing function of the nominal rate of interest (the rate of money growth).

When  $I > \hat{I}$ , the equilibrium rate of growth is a decreasing function of the nominal rate of interest (the rate of money growth). This function is strictly decreasing if savings are interest elastic ( $\theta > 0$ ).

Intuitively, a higher rate of money growth leads to a higher nominal rate of interest. When the incentive constraint is not binding ( $I \leq \hat{I}$ ), this higher nominal interest rate causes banks to economize on reserve holdings. The result is a change in the composition of bank portfolios that leads to more capital investment, and to higher rates of capital accumulation. This, of course, is a balanced growth version of the Mundell-Tobin effect.<sup>21</sup> Moreover, it bears emphasis that higher nominal interest rates are associated with higher rates of growth *even though higher nominal rates of interest reduce the savings rate of young agents*. The portfolio composition effect dominates the effect on the savings rate. Thus the adverse growth effects of low nominal rates of interest and low rates of inflation derive entirely from their implication for bank portfolio allocations. This adverse portfolio effect reduces capital investment despite the fact that the savings rate is maximized at  $I=1$ .<sup>22</sup>

On the other hand, when the incentive constraint is binding ( $I > \hat{I}$ ), increases in the nominal rate of interest (the rate of inflation) make the credit market friction more severe. As a result, banks ration credit more heavily and increases in the nominal rate of interest cause banks to reduce the share of loans in their portfolios. This portfolio composition effect is compounded by a reduced savings rate, implying an unambiguous reduction in the real rate of growth.

The equilibrium growth rate, for all  $I > 1$ , is depicted in Figure 4. Notice that this figure is consistent with a substantial body of empirical evidence suggesting that there exist threshold effects associated with the long-run rate of inflation.<sup>23</sup> In particular, for inflation rates (or rates of money growth) below some threshold, permanent increases in the rate of inflation are associated with increases in the long-run rate of real growth. However once inflation exceeds some threshold level, further increases in it actually cause growth to decline. Our analysis offers one explanation as to why this might be the case.

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<sup>21</sup> See Mundell (1963) and Tobin (1965).

<sup>22</sup> If  $\rho > 1$  [ $\gamma(I) > 0$ ] holds, then higher nominal interest rates lead banks to expand their holdings of cash reserves, and reduce their investments in capital accumulation. As a result, higher nominal rates of interest can be associated with lower rates of real growth.

<sup>23</sup> See, for instance, Bullard and Keating (1995) or Khan and Senhadji (2000).

## B. Zero Nominal Interest Rates ( $I_t = 1$ ).

We now turn our attention to equilibria where the nominal rate of interest is zero. We begin with a discussion of balanced growth paths.

When nominal rates of interest are zero,  $R = p_t/p_{t+1} = m_{t+1}/\sigma m_t$ . Moreover, the bank balance sheet constraint (6), along with the government budget constraint (3), implies that

$$(29) \quad k_{t+1} + m_t = \eta(1) \left( (1-\alpha)Ak_t + \frac{\sigma-1}{\sigma} m_t \right); \quad t \geq 0.$$

Finally, along a balanced growth path,  $k_{t+1}/k_t = m_{t+1}/m_t = \sigma R$ . Using this fact in (29) allows one to obtain

$$(30) \quad m_t = \frac{\eta(1)(1-\alpha)A - \sigma R}{1 - \frac{\sigma-1}{\sigma}\eta(1)} k_t; \quad t \geq 0.$$

This set of conditions completely determines a balanced growth path equilibrium with zero nominal rates of interest.

Clearly several conditions must be satisfied in order for such an equilibrium to exist. One is that, with zero nominal rates of interest, banks wish to provide complete insurance against the risk of relocation. However, it is feasible for them to do so only if  $m_t \geq \pi s_t$  holds. Using (3), (2) and (30), the savings of a young agent can be written as

$$s_t = \eta(1) \left( w_t + \frac{\sigma-1}{\sigma} m_t \right) = \eta(1) \frac{(1-\alpha)A - (\sigma-1)R}{1 - \frac{\sigma-1}{\sigma}\eta(1)} k_t.$$

Then it is easy to verify that  $m_t \geq \pi s_t$  is satisfied iff

$$\sigma \leq \frac{\eta(1)((1-\alpha)A(1-\pi) - \pi R)}{R(1-\pi\eta(1))} \equiv \sigma(1).$$

Thus, in order for the nominal rate of interest to be zero,  $\sigma \leq \sigma(1)$  must hold. Once again, choosing  $\sigma \leq \sigma(1)$  is feasible only if  $\sigma(1) > 0$ .

We now have a complete characterization of balanced growth path equilibria. There exists a unique equilibrium displaying balanced growth and a positive nominal interest rate if  $\sigma(1) \leq 0$ , or if  $\sigma(1) > 0$  and  $\sigma > \sigma(1)$ . There is a unique equilibrium displaying balanced growth with  $I=1$ , if  $\sigma(1) > 0$  and  $\sigma \leq \sigma(1)$ . Credit rationing occurs (banks face a binding incentive constraint) iff  $\sigma > \sigma(\hat{I})$ . Finally, we note that, along a balanced growth path displaying a zero nominal rate of interest, the maximal rate of growth is achieved by setting the rate of money creation as high as possible: that is, by setting  $\sigma = \sigma(1)$ . The implied rate of real growth is then  $R\sigma(1) = \mu(1) \leq \mu(I)$ ,  $\forall I \in [1, \hat{I}]$ .

## C. Dynamics

In this section we investigate whether our economy has dynamical equilibria other than the balanced growth path equilibrium.

We begin by considering potential equilibria with  $I_t > 1$  for  $\forall t$ . In this case, the evolution of the capital stock, real balances and the nominal interest rate is governed by equations (20), (21) and (22). Combining these equations yields the following law of motion for the nominal interest rate:

$$(31) \quad I_t \frac{1 - \hat{\gamma}(I_t)}{\hat{\gamma}(I_t)} = \frac{\sigma R}{(1 - \alpha)A} \left[ \frac{1}{\hat{\gamma}(I_{t+1})\hat{\eta}(I_{t+1})} - \frac{\sigma - 1}{\sigma} \right]; \quad t \geq 0.$$

When  $I_t = I_{t+1}$ , (31) collapses to (24), implying that (31) has a unique steady state. We also know that in this steady state the incentive constraint does not bind if  $\sigma \leq \sigma(\hat{I})$ , and it binds if  $\sigma > \sigma(\hat{I})$ . We now characterize dynamics in a finite interval around the steady state.

First, we suppose that  $\sigma \leq \sigma(\hat{I})$ , and consider potential equilibria with  $I_t \in (1, \hat{I}]$ , for  $\forall t$ . In this case,  $\hat{\gamma}(I) = \gamma(I)$ , and (11) can be used to rewrite (31) as

$$(32) \quad \frac{1}{\psi(I_{t+1})} = 1 + \frac{1}{\sigma} \left[ \frac{(1 - \pi)(1 - \alpha)A}{\pi R} \cdot (I_t)^{\frac{1}{\rho}} - 1 \right],$$

where  $\psi(I) = \gamma(I)\eta(I)$ . Some properties of (32) are stated in the next lemma, which is proved in Appendix B.

*Lemma 2.* Assume that  $\rho < 1$  and  $\sigma \in (\max\{0, \sigma(1)\}, \sigma(\hat{I})]$ . Then (32) implicitly defines a unique law of motion for  $I_t$ , which satisfies  $dI_{t+1}/dI_t > 0$ . Moreover, at the steady state,  $dI_{t+1}/dI_t > 1$ .

The lemma establishes that the equilibrium law of motion for  $I_t$  has the configuration depicted in panel (a) of Figure 5. It follows that the dynamics of (32) is globally unstable in the interval  $(1, \hat{I}]$ . Therefore, the only dynamical equilibrium that satisfies  $I_t \in (1, \hat{I}]$  for  $\forall t$ , is the balanced growth path characterized earlier.<sup>24</sup>

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<sup>24</sup> If  $\rho > 1$  holds, then the equilibrium law of motion described by (32) is negatively sloped. The unique balanced growth path equilibrium may be either asymptotically stable or unstable. Thus there may be many dynamical equilibrium paths consistent with perpetually positive and bounded nominal rates of interest. In addition, all such equilibria — other than the one with a constant nominal interest rate — will display oscillation, so that indeterminacy of equilibrium and endogenous volatility will emerge. It is also possible that equilibria displaying two-period cycles will exist, so that endogenously arising volatility need not vanish asymptotically.

Second, we suppose that  $\sigma > \sigma(\hat{I})$ , and consider potential equilibria with  $I_t \in (\hat{I}, R/r]$ , for  $\forall t$ . In this case,  $\hat{\gamma}(I) = \tilde{\gamma}(I)$ , and (18) can be used to show that the law of motion given by (31) simplifies to

$$(33) \quad \frac{1}{\tilde{\Psi}(I_{t+1})} = 1 + \frac{1}{\sigma} \left[ \frac{(1-\alpha)A}{r\pi} - 1 \right],$$

where  $\tilde{\Psi}(I) = \tilde{\gamma}(I)\tilde{\eta}(I)$ . Clearly, equation (33) has only trivial associated dynamics. Every dynamical equilibrium that satisfies  $I_t \in (\hat{I}, R/r]$ , for  $\forall t$ , converges to the balanced growth path in at most one step.

We continue by considering dynamical equilibria that satisfy  $I_t = 1$  for  $\forall t$ . In this case,  $m_t/m_{t-1} = \sigma R \forall t$ , and equation (29) can be written as

$$(34) \quad \frac{k_{t+1}}{m_t} = \frac{\eta(1)(1-\alpha)A}{\sigma R} \frac{k_t}{m_{t-1}} - \left[ 1 - \frac{\sigma-1}{\sigma} \eta(1) \right]; \quad t \geq 0.$$

A valid equilibrium must obey

$$(35) \quad 0 \leq k_{t+1}/m_t \leq (1-\pi)/\pi,$$

where the second inequality follows from the requirement that  $m_t \geq \pi s_t$ . If  $\sigma \in [0, \sigma(1)]$ , then it is readily established that  $\eta(1)(1-\alpha)A > \sigma R$ , and hence the unique balanced growth path is unstable. The law of motion described by equation (34) is depicted in panel (b) of Figure 5. Clearly, any candidate non-stationary equilibrium must violate (35) after a finite number of periods. Thus, the only possible equilibrium with a zero nominal rate of interest is the balanced growth path derived above, along which  $k_{t+1}/m_t$  is constant.

Finally, we remark that in order to completely characterize the global dynamics of our economy, we would have to consider equilibria where over time  $I_t$  is allowed to move between the categories  $I_t=1$ ,  $I_t \in (1, \hat{I}]$ , and  $I_t \in (\hat{I}, R/r]$ . It is tedious, but possible to show that such “regime switching” equilibria are also non-existent.<sup>25</sup>

In conclusion, we have established that for any value of  $\sigma \in (0, \hat{\sigma}(R/r)]$  the economy has a unique dynamical equilibrium, which features a constant nominal interest rate and balanced growth.

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<sup>25</sup> The key step in the analysis of such equilibria is the choice of a sufficient state variable that works across all three regimes. A suitable variable, for example, is  $I_t/\gamma_t$ . Further details of the proof are omitted.



## V. Welfare

We now wish to evaluate the optimal rate of money growth along a balanced growth path. We are particularly interested to know two things: (a) whether such a rate of money creation implies positive or zero nominal rates of interest, and (b) whether such a rate of money growth implies that credit is or is not rationed.

Choosing different money growth rates affects the economy through two main channels. First, higher rates of money creation (higher nominal rates of interest) are associated with more rapid rates of real growth, at least as long as  $\sigma < \sigma(\hat{I})$ . Second, positive nominal interest rates cause banks to perceive an opportunity cost of holding cash balances, which interferes with the provision of insurance against the risk of relocation. An optimizing government must confront this trade-off. Intuitively, this logic suggests that there is little reason for an optimizing government to choose a money growth rate below  $\sigma(1)$  or above  $\sigma(\hat{I})$ . Both of these choices would reduce the real rate of growth without improving insurance provision. However, in order to formalize this argument, we also need to consider how different rates of money creation affect the value of transfers received by young agents.

We begin by considering the government's trade-offs when  $I \in (1, \hat{I}]$ . We then show that the government will never want to set  $I > \hat{I}$ , so that in an optimum credit is never rationed. Finally, we consider the government's optimal policy among the set of policies consistent with a zero nominal rate of interest. Having done so, it will be possible to state conditions under which the Friedman rule is and is not optimal.

### A. Positive Nominal Interest Rates and a Non-binding Incentive Constraint

Clearly we must begin by ascribing some objective function to the government. We take the government's objective function to be the discounted sum of the expected utilities of all current and future young generations, where the government discounts the future at the rate  $\beta < 1$ .<sup>26</sup>

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<sup>26</sup> Thus the government does not take account of the welfare of the initial old generation. We describe below how the analysis would need to be modified if the government also considered the welfare of this generation.

We have already shown that when  $1 < I < \hat{I}$  holds, the (maximized) expected utility of a representative member of the generation born at  $t$  is given by the expression  $\frac{\theta}{1-\rho}(w_t + \tau_t)^{1-\rho} [1 - \eta(I_t)]^{-\rho}$ . Furthermore, if we define the function  $\chi(I)$  by

$$\chi(I) \equiv \left[ 1 - \frac{\sigma(I)-1}{\sigma(I)} \gamma(I) \eta(I) \right]^{-1}; \quad I \in (1, \hat{I}],$$

then we also have that  $w_t + \tau_t = \chi(I)(1-\alpha)Ak_t$  and  $k_t = (\mu(I))^t k_0$ , along a balanced growth path. Thus, the welfare of a member of generation  $t$ , as a function of the equilibrium nominal rate of interest  $I$ , is

$$\frac{\theta}{1-\rho} \left[ \chi(I) \mu(I)^t \right]^{1-\rho} [1 - \eta(I)]^{-\rho} [(1-\alpha)Ak_0]^{1-\rho}.$$

It is also easy to verify that this expression gives the welfare of a representative member of generation  $t$  when  $\sigma = \sigma(1)$  holds.

We can now view the government as choosing a value for the nominal rate of interest,<sup>27</sup>  $I \in [1, \hat{I}]$ , to maximize

$$\sum_{t=0}^{\infty} \beta^t (1 - \eta(I))^{-\rho} \left[ \chi(I) \mu(I)^t \right]^{1-\rho} = \frac{(1 - \eta(I))^{-\rho} \chi(I)^{1-\rho}}{1 - \beta(\mu(I))^{1-\rho}} \equiv \Omega(I),$$

so that  $\Omega(I)$  is the government's objective function. In order for the function  $\Omega$  to be well-defined, we must have  $\beta\mu(I)^{1-\rho} < 1, \forall I$ . It is easy to check that this condition is satisfied if

$$(a.1) \quad \beta[\eta(1)(1-\alpha)A]^{1-\rho} < 1,$$

as we henceforth assume. Finally, let  $I^* = \operatorname{argmax} \Omega(I); I \in [1, \hat{I}]$

The following proposition states our results about the sub-optimality of the Friedman rule. Its proof appears in Appendix C.

Proposition 2.

(a) Suppose that  $\sigma(1) > 0$ , and that

$$(36) \quad R > \mu(1)(1 - \pi\eta(1)) \left[ 1 - \beta(\mu(1))^{1-\rho} \right]$$

is satisfied. Then it is feasible, but not optimal, to follow the Friedman rule. The optimal choice of  $I$  satisfies  $I^* > I$ .

(b) A sufficient condition for the Friedman rule to be sub-optimal is that

$$R \geq \mu(1).$$

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<sup>27</sup> Once the optimal nominal rate of interest is chosen, the money growth rate is given by  $\sigma(I)$ .

The first inequality in part (a) of the proposition states a condition under which welfare can be increased by raising the nominal interest rate above zero. Not surprisingly, this inequality will be satisfied whenever setting  $I=1$  leads to a sufficiently low rate of real growth. Furthermore, note that  $\beta$  governs the extent to which the government is willing to trade off liquidity provision for growth. For higher values of  $\beta$ , condition (36) is more likely to be satisfied, meaning that if the government cares about future generations more, it is more likely to want to stimulate growth by driving the nominal rate of interest above zero.<sup>28</sup>

Part (b) of the proposition states that the Friedman rule *cannot* be optimal whenever the maximal rate of growth associated with a zero nominal rate of interest is below the real rate of interest,  $R$ . This finding reflects some well-known results about golden rule allocations in conventional overlapping generations models with production. Indeed, when such models have steady states, steady state welfare is increased by promoting capital accumulation whenever the real rate of interest exceeds the rate of growth. The same kind of reasoning clearly obtains in this context as well.<sup>29</sup> We note that this reasoning does not depend on how risk averse agents may be. This follows from the fact that, when  $I=1$  holds, agents receive full insurance against the risk of relocation, and therefore they are locally risk neutral. Thus the Friedman rule is not optimal, and the government should raise the nominal rate of interest, whenever the real rate of interest exceeds the real rate of growth at  $I=1$ . Moreover, it bears emphasis that this is true no matter how small  $\beta$  is; that is, when  $R \geq \mu(1)$ , the Friedman rule will not be optimal even if almost no weight is attached to the utility of generations after the first.<sup>30</sup>

Finally, we emphasize that the sub-optimality of the Friedman rule is not caused by the production externality. Because of the externality, private agents perceive real rates of return on savings that are below the social marginal product of capital. In this model, this may or may not lead agents to save “too little”. If agents do save too little, the only way in which monetary policy can be used to increase savings is to keep the nominal rate of interest low. Hence, if

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<sup>28</sup> If  $\rho > 1$  holds, then the Friedman rule is suboptimal if the inequality in equation (36) is reversed.

<sup>29</sup> How would this analysis be modified if the government also cared about the welfare of the initial old generation? The consumption, and hence the welfare of this generation is affected by the choice of  $I$  only through the effect of  $I$  on the value of initial real balances. This value is easily shown to be given by  $\gamma(I)/\sigma(I)$ . Thus, if the government weights the welfare of the initial old, we can represent this by appending the constraint  $\gamma(I)/\sigma(I) \geq V$  to the government’s problem, where  $V$  depends on the minimum welfare level to be offered to the initial old. If this constraint is not binding at the value  $I=1$ , it continues to be the case that the Friedman rule is not optimal.

<sup>30</sup> If  $\beta=0$ , the government’s objective function becomes the utility of the first generation. Since the initial capital stock (and therefore the initial wage rate) is given, monetary policy affects welfare only by affecting insurance provision and the value of transfers. Consequently, in this extreme case, it is the higher value of transfers that compensates agents for the suboptimal insurance provision implied by deviating from the Friedman rule.

anything, the presence of the production externality introduces a bias in the direction if the Friedman rule being optimal.

It remains to say more about what the optimal choice  $I^*$  actually is, at least over the interval  $[1, \hat{I}]$ . Proposition 4 gives some results on this point. Its proof appears in Appendix D.

Proposition 3.

(a) Suppose that the conditions of Proposition 3 (a) and

$$(37) \quad \beta(\mu(1))^{1-\rho} \geq \frac{1+\rho}{1+2\rho-\rho^2}$$

are satisfied. Then  $I^* = \hat{I}$ .

(b) Suppose that the conditions of Proposition 3 (a) hold, that

$$(38) \quad \beta(\mu(\hat{I}))^{1-\rho} < \frac{1+\rho}{1+2\rho-\rho^2},$$

and that  $1 \geq (2-\rho)(\mu(\hat{I}))^{1-\rho}$  all hold. Then the optimal choice of a nominal rate of interest satisfies  $I^* \in (1, \hat{I})$  if

$$(39) \quad R^{1-\rho}((1-\pi)r)^\rho \leq \mu(1)(1-\beta(\mu(1))^{1-\rho})(1-\pi\eta(1)).$$

(c) Suppose that the conditions of Proposition 3 (a) hold, that (38) holds, and that  $1 < (2-\rho)\beta(\mu(1))^{1-\rho}$ . Then the optimal value of  $I$  satisfies  $I^* \in (1, \hat{I})$  if

$$(40) \quad R^{1-\rho}((1-\pi)r)^\rho \leq \mu(\hat{I})(1-\beta(\mu(\hat{I}))^{1-\rho})(1-\pi\eta(1)).$$

Proposition 4 states conditions under which the gains in real growth that derive from driving the nominal rate of interest to  $\hat{I}$  (the growth maximizing nominal rate of interest) are and are not large enough to overcome the associated losses in risk sharing that occur as banks economize to a greater and greater degree on their holdings of reserves.

## B. Positive Nominal Interest Rates and Credit Rationing

Intuitively, there seems to be no reason for the government to increase the nominal rate of interest above the level  $\hat{I}$ . Doing so does not stimulate growth, and it interferes with the provision of liquidity (insurance) by banks. However, because of the fact that money is injected via lump-sum transfers here, the possibility exists that raising the rate of money growth above  $\sigma(\hat{I})$  increases the value of the government's objective function. This could occur if the value of the transfers received by young agents was enough to more than outweigh the other two considerations. We now demonstrate that this is not the case, and that — in fact — the optimal nominal rate of interest never exceeds  $\hat{I}$ .

When the incentive constraint (10) is binding in banks' problems, we have already shown that the expected utility of a young agent born at  $t$  is  $\frac{\theta}{1-\rho}(w_t + \tau_t)^{1-\rho}(1-\tilde{\eta}(I))^{-\rho}$ . We have also demonstrated that  $w_t + \tau_t = \tilde{\chi}(I)(1-\alpha)Ak_t$ , where

$$\tilde{\chi}(I) \equiv \left[ 1 - \tilde{\eta}(I)\tilde{\gamma}(I) + \frac{\tilde{\eta}(I)\tilde{\gamma}(I)}{\tilde{\sigma}(I)} \right]^{-1} = 1 - \frac{\frac{\pi R}{(1-\alpha)A}}{1 - \tilde{\eta}(I)\tilde{\gamma}(I)}.$$

Thus the expected utility of a young agent born at  $t$  is given by the expression

$$\frac{\theta}{1-\rho}((1-\alpha)Ak_0)^{1-\rho} \left[ \tilde{\chi}(I)\tilde{\mu}(I)^t \right]^{1-\rho} (1-\tilde{\eta}(I))^{-\rho}.$$

If the government discounts the utility of future generations at the rate  $\beta$ , then over the range  $I > \hat{I}$ , the government's objective function is<sup>31</sup>

$$\sum_{t=0}^{\infty} \beta^t (1-\tilde{\eta}(I))^{-\rho} \left[ \tilde{\chi}(I)\tilde{\mu}(I)^t \right]^{1-\rho} = \frac{(1-\tilde{\eta}(I))^{-\rho} \tilde{\chi}(I)^{1-\rho}}{1-\beta(\tilde{\mu}(I))^{1-\rho}} \equiv \tilde{\Omega}(I).$$

We now state the following result.<sup>32</sup>

**Proposition 4.** For all  $I \in (\hat{I}, R/r)$ ,  $\tilde{\Omega}'(I) \leq 0$  holds. The inequality is strict if  $\theta > 0$ .

The proof of Proposition 5 appears in Appendix E. The proposition asserts that young agents do not benefit sufficiently from higher lump-sum transfers to overturn the fact that increasing the nominal rate of interest above  $\hat{I}$  interferes with both real growth, and the provision of liquidity by banks.

## C. Zero Nominal Rates of Interest

It remains to consider the possibility that it is optimal for the government to set  $\sigma < \sigma(1)$ . Intuitively one would expect that this choice cannot be optimal. Setting  $\sigma = \sigma(1)$  allows banks to provide complete insurance against the risk of relocation, and it maximizes the rate of real growth that is attainable with a zero nominal rate of interest. However, again the fact that money is injected via lump-sum transfers to young agents means that the consequences of these transfers must be considered in evaluating the government's objective function.

<sup>31</sup> Assumption (a.1) continues to imply that the government's objective function is well-defined for all  $I \in (\hat{I}, R/r]$ .

<sup>32</sup> If  $\rho > 1$  holds, then it is possible that  $\tilde{\Omega}'(I) > 0$  holds for some  $I > \hat{I}$  satisfying (8). Whether this condition holds or not depends on the magnitude of  $\beta$ . Thus it need not be the case that the optimal choice of the nominal rate of interest is less than or equal to  $\hat{I}$ .

We have already demonstrated that, when nominal rates of interest are zero, the expected utility of a young agent born at  $t$  equals  $\frac{\theta}{1-\rho}(w_t + \tau_t)^{1-\rho} [1 + \theta^{-1/\rho} R^{(1-\rho)/\rho}]^\rho$ . In addition, equation (33) implies that  $w_t + \tau_t = G(\sigma)k_t$ , where

$$G(\sigma) \equiv \sigma \frac{(1-\alpha)A - (\sigma-1)R}{\sigma - (\sigma-1)\eta(1)}.$$

We now state the following result. Its proof is given in Appendix F.

*Lemma 3.* Suppose that  $R \geq \mu(1)$ . Then  $G'(\sigma) \geq 0$ , for all  $\sigma \leq \sigma(1)$ .

Lemma 4 states a simple condition under which, in the interval  $(0, \sigma(1)]$ , the value of a young agent's lump-sum transfer is always increased by increasing  $\sigma$ . Since, in this interval, increasing the rate of money growth also increases the rate of real growth, and since it does not interfere with insurance provision, it is therefore not optimal to set  $\sigma < \sigma(1)$  if the real rate of interest is greater than or equal to the maximal rate of real growth consistent with a zero nominal rate of interest. This result, together with Proposition 3, then implies that the Friedman rule is sub-optimal. Or, in other words,  $R \geq \mu(1)$  is a sufficient (although far from a necessary) condition for it to be desirable to have positive nominal rates of interest in this economy.

## VI. Conclusions

A large literature states conditions under which it is optimal for a monetary authority to drive the nominal rate of interest to zero. Doing so equates the social cost of creating outside money with agents' perceptions of the opportunity cost of holding it. Nonetheless, experience suggests that having nominal rates of interest at or near zero need not lead to desirable outcomes. In fact, the closest practical approximations to the Friedman rule have been observed in places like the U.S. during the Great Depression, or in Japan recently. The result has invariably been a severe and long-lasting recession.

This paper has pursued the notion that low nominal rates of interest can have very negative implications for real growth. In particular, when nominal rates of interest are (nearly) zero, money is a very good asset. As a result, banks have limited incentives to lend. The consequence is low rates of capital investment, and low rates of real growth. And, indeed, in situations like the Great Depression — or like that in Japan today — not only have real rates of growth been very low, but so have levels of bank lending to the private sector and rates of capital

investment. As we have noted, low nominal interest rates can lead to this outcome even when they raise overall savings rates.

When the maximal rate of real growth consistent with a zero nominal rate of interest is less than or equal to the real rate of interest, it will be desirable for the government to raise the rate of money creation, and the nominal rate of interest, in order to promote real growth. This is true independently of the rate at which the government discounts the utility of future generations. Nor does this result depend in any particular way on the endogenous growth formulation we have employed. Analogous results can be obtained regarding the relation between the nominal rate of interest and the steady state capital stock in economies that do not allow for sustained growth.

Of course there is a limit on the extent to which money creation can be used to promote growth. Considerable empirical evidence suggests that higher long-run rates of money creation can promote long-run real growth, over some range. But, this same evidence suggests that, once the rate of money creation exceeds some threshold level, further increases in the money growth rate (the rate of inflation) are detrimental to long-run real performance. Our analysis is consistent with this finding as well. And, we have stated conditions under which a benevolent government will and will not want to push the rate of money creation to its growth maximizing level.

Naturally our analysis has abstracted from a number of issues. One is the possibility that the government has revenue needs. Introducing a sequence of government expenditures would allow us to consider whether or not the government's incentives to print money would be substantially altered by the possibility of using inflationary finance. The optimal use of such finance here would probably differ significantly from that in the existing literature on the Friedman rule (see footnote 4). And, it would raise the possibility that the government would want to regulate banks as part of an optimal tax scheme in order to enhance the inflation tax base.<sup>33</sup> Another obvious extension of the analysis would be to allow the government to issue (potentially) interest bearing bonds as well as money. If seigniorage income can be used to pay interest on government bonds, the nature of the relationship between inflation and real rates of growth can be substantially different from the one demonstrated above.<sup>34</sup> It would be interesting to see how these modifications of the analysis would affect the optimality of the Friedman rule.

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<sup>33</sup> See, for instance, Bencivenga and Smith (1992), Bhattacharya, et. al. (1997), or Espinosa and Yip (2000).

<sup>34</sup> This point is discussed by Schreft and Smith (1997).

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## Appendix

### A. Proof of Lemma 1.

Part (a) of the lemma follows immediately from the definition of  $\sigma(I)$  in equation (26), and the fact that  $\chi(1)=\pi$ . Part (b) is implied by part (c). Part (c) can be proved as follows. If  $I \leq \hat{I}$ , then

$$(A.1) \quad \frac{I\sigma'(I)}{\sigma(I)} = \frac{I\psi'(I)}{\psi(I)} \frac{1}{1-\psi(I)} + \frac{\rho^{-1}(1-\alpha)A(1-\pi)I^{1/\rho}}{(1-\alpha)A(1-\pi)I^{1/\rho} - \pi R},$$

where  $\psi(I) \equiv \gamma(I)\eta(I)$ . Furthermore,

$$(A.2) \quad \frac{I\psi'(I)}{\psi(I)} = \frac{I\gamma'(I)}{\gamma(I)} + \frac{I\eta'(I)}{\eta(I)} = -\frac{1-\rho}{\rho}(1-\gamma(I)) - \frac{1-\rho}{\rho}\gamma(I)(1-\eta(I)) = -\frac{1-\rho}{\rho}(1-\psi(I)).$$

Substituting (A.2) into (A.1) and rearranging terms yields

$$(A.3) \quad \frac{I\sigma'(I)}{\sigma(I)} = 1 + \rho^{-1} \frac{\pi R}{(1-\alpha)A(1-\pi)I^{1/\rho} - \pi R} > 1.$$

If  $I \leq \hat{I}$ , then

$$(A.4) \quad \frac{I\tilde{\sigma}'(I)}{\tilde{\sigma}(I)} = \frac{I\tilde{\psi}'(I)}{\tilde{\psi}(I)} \frac{1}{1-\tilde{\psi}(I)},$$

where, as before, we define  $\tilde{\psi}(I) = \tilde{\gamma}(I)\tilde{\eta}(I)$ . In addition, it is easy to show that

$$(A.5) \quad \frac{I\tilde{\psi}'(I)}{\tilde{\psi}(I)} = 1 - \tilde{\psi}(I) - \rho^{-1}(\tilde{\gamma}(I) - \tilde{\gamma}(I)\tilde{\eta}(I)).$$

Substituting (A.5) into (A.4) we obtain

$$(A.6) \quad \frac{I\tilde{\sigma}'(I)}{\tilde{\sigma}(I)} = 1 - \rho^{-1} \frac{\tilde{\gamma}(I)(1-\tilde{\eta}(I))}{1-\tilde{\psi}(I)} \leq 1.$$

Strict inequality obtains iff  $\tilde{\eta}(I) < 1$ , which occurs iff  $\theta > 0$ .

### B. Proof of Lemma 2

Since  $\gamma'(I) < 0$  and  $\eta'(I) < 0$ , it follows that  $\psi'(I) < 0$ , so that the left hand side of (32) is strictly increasing in  $I_{t+1}$ . Therefore, if (32) can be solved for  $I_{t+1}$ , the solution is unique. Applying the implicit function theorem to (32) results in

$$(A.7) \quad \frac{dI_{t+1}}{dI_t} = -\frac{1}{\rho} \cdot \frac{(1-\pi)(1-\alpha)A}{\pi\sigma R} (I_t)^{\frac{1}{\rho}} \psi(I_{t+1}) \frac{I_{t+1}}{I_t} \frac{\psi(I_{t+1})}{I_{t+1}\psi'(I_{t+1})}$$

Substituting (A.2) and (32) into (A.7), we get

$$(A.8) \quad \frac{I_t}{I_{t+1}} \frac{dI_{t+1}}{dI_t} = (1-\rho)^{-1} \left[ \frac{1 - \frac{\sigma-1}{\sigma} \psi(I_{t+1})}{1 - \psi(I_{t+1})} \right].$$

Clearly, (A.8) implies that when  $I_{t+1}=I_t$ ,  $dI_{t+1}/dI_t > 1$ .

## C. Proof of Proposition 2.

In order to prove Proposition 2, it will be useful to begin with the following lemma.

Lemma A.1. The function  $\chi(I)$  has the representation

$$\chi(I) = \frac{\mu(I)}{(1-\alpha)A\eta(I)(1-\gamma(I))}.$$

Proof of Lemma A.1. The definitions of the functions  $\chi(I)$  and  $\sigma(I)$  imply that

$$\begin{aligned} \chi(I) &= \frac{1}{1 - \frac{\sigma-1}{\sigma} \gamma(I)\eta(I)} = \frac{(1-\alpha)A(1-\pi)I^{1/\rho} - \pi R}{(1-\gamma(I)\eta(I))((1-\alpha)A(1-\pi)I^{1/\rho})} \\ &= \frac{I\mu(I)}{(1-\alpha)A^{\frac{1-\pi}{\pi}}\eta(I)\gamma(I)I^{1/\rho}} = \frac{\mu(I)}{(1-\alpha)A\eta(I)(1-\gamma(I))}, \end{aligned}$$

as claimed.

To continue with the proof of Proposition 2, define the function  $H(I)$  by

$$(A.9) \quad H(I) \equiv \left(1 - \beta(\mu(I))^{1-\rho}\right)^{-1}.$$

Then equation (A.9) and Lemma A.1 imply that the government's objective function takes the form

$$(A.10) \quad \Omega(I) = H(I)(1-\eta(I))^{-\rho} \left[ \frac{\mu(I)}{\eta(I)(1-\gamma(I))} \right]^{1-\rho}.$$

It then follows from (A.10) that

$$(A.11) \quad \frac{I\Omega'(I)}{\Omega(I)} = \frac{IH'(I)}{H(I)} + \rho \frac{I\eta'(I)}{1-\eta(I)} - (1-\rho) \left[ \frac{I\eta'(I)}{\eta(I)} - \frac{I\mu'(I)}{\mu(I)} - \frac{I\gamma'(I)}{1-\gamma(I)} \right].$$

Moreover, we observe that

$$(A.12) \quad \frac{I\gamma'(I)}{1-\gamma(I)} = -\frac{1-\rho}{\rho} \gamma(I),$$

$$(A.13) \quad \frac{IH'(I)}{H(I)} = \frac{(1-\rho)\beta(\mu(I))^{1-\rho}}{1-\beta(\mu(I))^{1-\rho}} \frac{I\mu'(I)}{\mu(I)}$$

$$(A.14) \quad \frac{I\eta'(I)}{1-\eta(I)} = -\frac{1-\rho}{\rho}\gamma(I)(1-\eta(I)),$$

and

$$(A.15) \quad \frac{I\mu'(I)}{\mu(I)} = \frac{R\gamma(I)\eta(I)}{\rho I\mu(I)(1-\gamma(I)\eta(I))}.$$

Substituting equations (A.12)–(A.15) into (A.11) and rearranging terms yields

$$\frac{I\Omega'(I)}{\Omega(I)} = \frac{1-\rho}{\rho}\gamma(I)\eta(I) \left[ \frac{R}{I(1-\gamma(I)\eta(I))\mu(I)(1-\beta(\mu(I))^{1-\rho})} - 1 \right].$$

It follows that  $\Omega'(1) > 0$  holds iff

$$(A.16) \quad R > \mu(1)(1-\pi\eta(1))(1-\beta(\mu(1))^{1-\rho}),$$

where (A.16) follows from the fact that  $\gamma(1) = \pi$ . This establishes part (a) of the proposition.

Part (b) is then immediate, since  $1 > (1-\pi\eta(1))(1-\beta\mu(1)^{1-\rho})$ .

### D. Proof of Proposition 3.

Define the function  $Q(I)$  by

$$(A.17) \quad Q(I) \equiv \frac{RH(I)}{I\mu(I)(1-\psi(I))},$$

where, as before,  $\psi(I) \equiv \gamma(I)\eta(I)$ , and where the function  $H(I)$  is defined in (A.9). Then Appendix C establishes that  $\Omega'(I) \geq 0$  holds iff  $Q(I) \geq 1$ . And, if the conditions of Proposition 2 (a) hold,  $\Omega'(1) > 0$  [ $Q(1) > 1$ ] is satisfied.

Differentiating (A.17) yields

$$(A.18) \quad \frac{IQ'(I)}{Q(I)} = \frac{IH'(I)}{H(I)} - 1 - \frac{I\mu'(I)}{\mu(I)} + \frac{I\psi'(I)}{\psi(I)} \frac{1}{1-\psi(I)}.$$

Substituting (A.2), (A.13) and (A.15) into (A.18) and rearranging terms, we obtain

$$\frac{IQ'(I)}{Q(I)} = \frac{\beta(1+2\rho-\rho^2)(\mu(I))^{1-\rho} - (1+\rho)}{\rho(1-\beta(\mu(I))^{1-\rho})}.$$

Thus, if

$$(A.19) \quad \beta(\mu(I))^{1-\rho} \geq \frac{1+\rho}{1+2\rho-\rho^2}$$

is satisfied for all  $I \in (1, \hat{I}]$ , and if the conditions of Proposition 2 (a) hold,  $Q(I) > 1$  is satisfied for all  $I \in [1, \hat{I}]$ . Moreover, (A.19) is satisfied for all  $I$  if (37) holds. It follows that  $I^* = \hat{I}$ . This establishes part (a) of the proposition.

For parts (b) and (c), satisfaction of (38) is required in order for  $Q(\hat{I}) < Q(1)$  to hold. Moreover,  $Q(\hat{I}) < 1$  [ $\Omega'(\hat{I}) < 0$ ] is satisfied iff

$$\mu(\hat{I})(1 - \psi(\hat{I})) \left( 1 - \beta(\mu(\hat{I}))^{1-\rho} \right) > \frac{R}{\hat{I}} \equiv R^{1-\rho} ((1-\pi)r)^\rho.$$

In addition, since  $\psi(I)$  is a decreasing function, it follows that  $1 - \psi(\hat{I}) > 1 - \psi(1) = 1 - \pi\eta(1)$ . Finally, it is straightforward to show that the term  $\mu(I) \left( 1 - \beta(\mu(I))^{1-\rho} \right)$  is increasing in  $I$  iff

$$(A.20) \quad 1 \geq (2-\rho)\beta(\mu(I))^{1-\rho}.$$

Thus if

$$(A.21) \quad \beta(\mu(I))^{1-\rho} > \frac{1+\rho}{1+2\rho-\rho^2}$$

and (A.20) are satisfied for all  $I \in (1, \hat{I}]$ , and if the conditions of Proposition 2 (a) hold, then equation (39) is sufficient for  $Q(\hat{I}) < 1$  [ $\Omega'(\hat{I}) < 0$ ] to hold. Moreover, (A.20) and (A.21) are satisfied for all  $I$  if  $1 \geq (2-\rho)\beta(\mu(\hat{I}))^{1-\rho}$  and (38) hold. Hence  $I^* < \hat{I}$  obtains. Similarly, if (A.21) holds but (A.20) is violated for all  $I \in (1, \hat{I}]$ , then equation (40) is sufficient for  $Q(\hat{I}) < 1$  [ $\Omega'(\hat{I}) < 0$ ] to be satisfied. Since  $1 < (2-\rho)\beta(\mu(1))^{1-\rho}$  implies that  $1 < (2-\rho)\beta(\mu(I))^{1-\rho}$  for all  $I$ , we again have  $I^* < \hat{I}$ . This establishes the proposition.

## E. Proof of Proposition 4.

Straightforward differentiation yields

$$(A.22) \quad \frac{I\tilde{\Omega}'(I)}{\tilde{\Omega}(I)} = \frac{(1-\rho)\beta(\tilde{\mu}(I))^{1-\rho}}{1-\beta(\tilde{\mu}(I))^{1-\rho}} \frac{I\tilde{\mu}'(I)}{\tilde{\mu}(I)} + \rho \frac{I\tilde{\eta}'(I)}{1-\tilde{\eta}(I)} + (1-\rho) \frac{I\tilde{\psi}'(I)}{1-\tilde{\psi}(I)},$$

where  $\tilde{\psi}(I) = \tilde{\gamma}(I)\tilde{\eta}(I)$ . Substituting (19) and (A.5) into (A.22) gives

$$\frac{I\tilde{\Omega}'(I)}{\tilde{\Omega}(I)} = \frac{(1-\rho)\beta(\tilde{\mu}(I))^{1-\rho}}{1-\beta(\tilde{\mu}(I))^{1-\rho}} \frac{I\tilde{\mu}'(I)}{\tilde{\mu}(I)} - (1-\rho)\tilde{\psi}(I) + (1-\rho)\tilde{\psi}(I) \left[ 1 - \rho^{-1} \frac{\tilde{\gamma}(I) - \tilde{\psi}(I)}{1-\tilde{\psi}(I)} \right]$$

Since  $\tilde{\mu}'(I) \leq 0$ , it follows that, for all  $I > \hat{I}$ ,  $\tilde{\Omega}'(I) \leq 0$ . Moreover, the inequality is strict if  $\theta > 0$ , so that  $\tilde{\psi}(I) \equiv \tilde{\gamma}(I)\tilde{\eta}(I) < \tilde{\gamma}(I)$ .

## F. Proof of Lemma 3.

Straightforward differentiation implies that

$$\frac{G'(\sigma)}{G(\sigma)} = \frac{\eta(1)}{1 - \frac{\sigma-1}{\sigma}\eta(1)} - \frac{R}{(1-\alpha)A - (\sigma-1)R}.$$

Note, then, that  $G'(\sigma) > 0$  necessarily holds if  $(\sigma-1)R > (1-\alpha)A$ . And, if  $(\sigma-1)R < (1-\alpha)A$  holds, then  $G'(\sigma) > 0$  is satisfied iff

$$(A.23) \quad \eta(1)(1-\alpha)A \geq \sigma^2 R \left(1 - \frac{\sigma-1}{\sigma} \eta(1)\right).$$

Since  $\sigma \leq \sigma(1)$ , note that a sufficient condition for (A.23) to obtain, and hence for  $G'(\sigma) > 0$  to hold, is that

$$(A.24) \quad \eta(1)(1-\alpha)A \geq R\sigma(1)^2.$$

Now, since  $\mu(I) \equiv \sigma(I)R/I$ , it follows that  $R\sigma(1)^2 \equiv \mu(1)^2/R$ . Moreover, it is easy to verify that  $\mu(I) \leq \eta(I)(1-\alpha)A \leq \eta(1)(1-\alpha)A$ , for all  $I$ . Then (A.24) is satisfied if

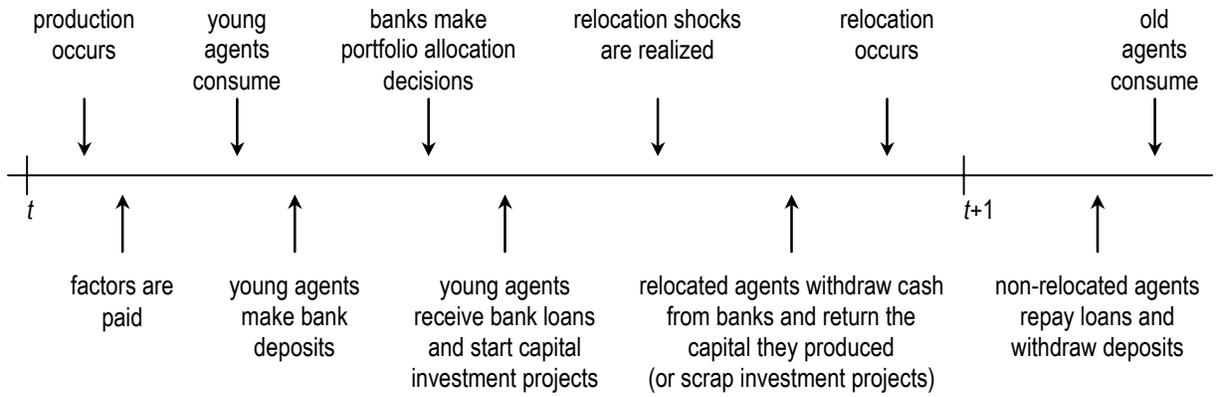
$$\eta(1)(1-\alpha)A \geq (\mu(1)/R)\mu(1).$$

But this necessarily holds if  $R \geq \mu(1)$ .

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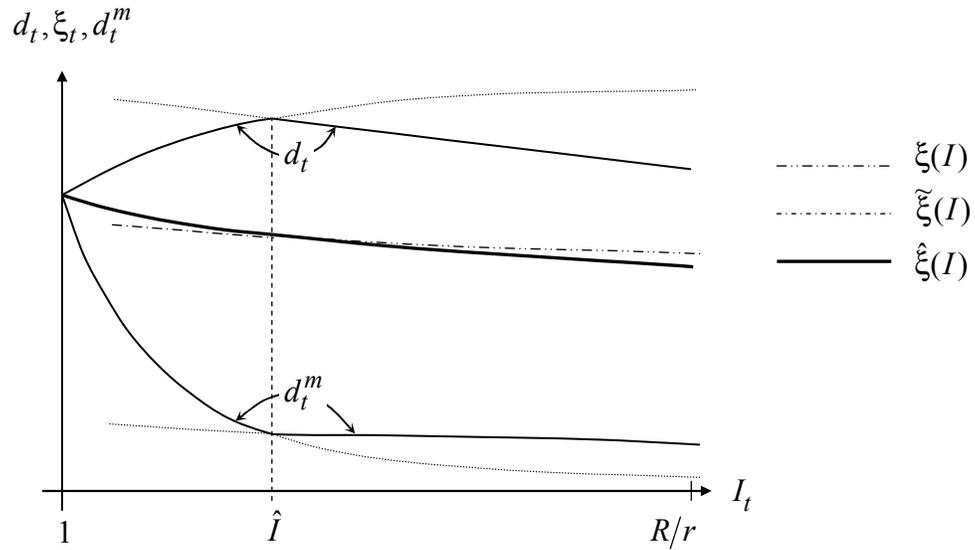
**Figure 1.**  
The timing of events

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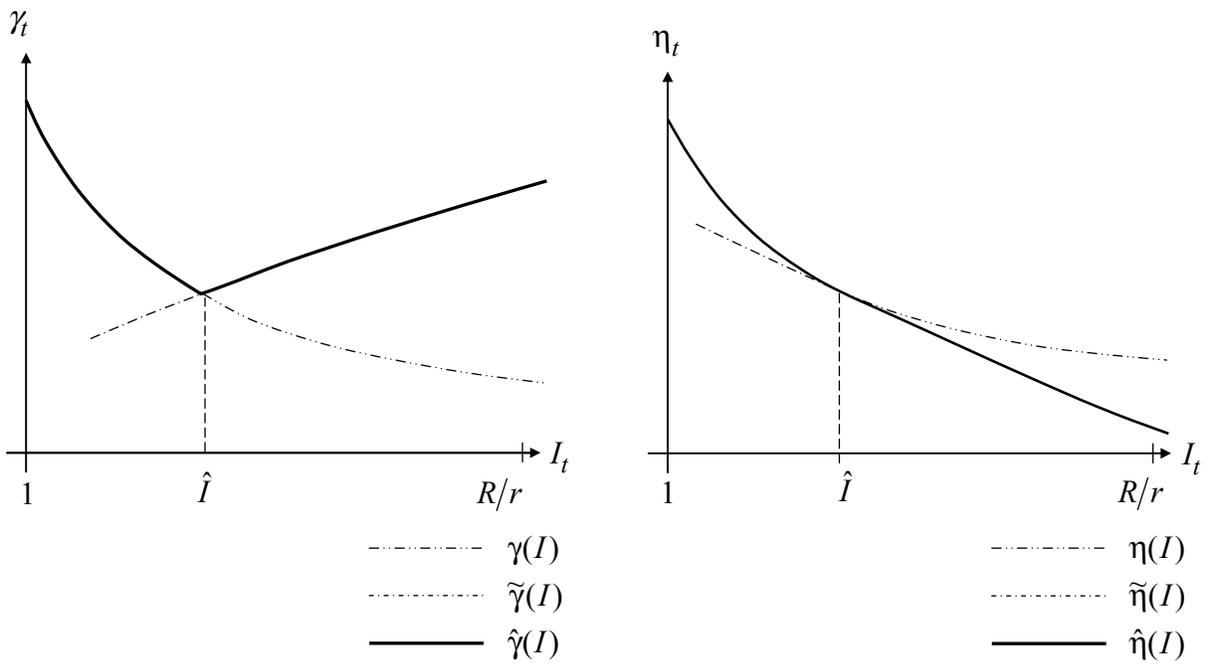
**Figure 2.**

Rates of returns received by movers and non-movers, and the certainty equivalent return on deposits as a function of the nominal interest rate.



**Figure 3.**

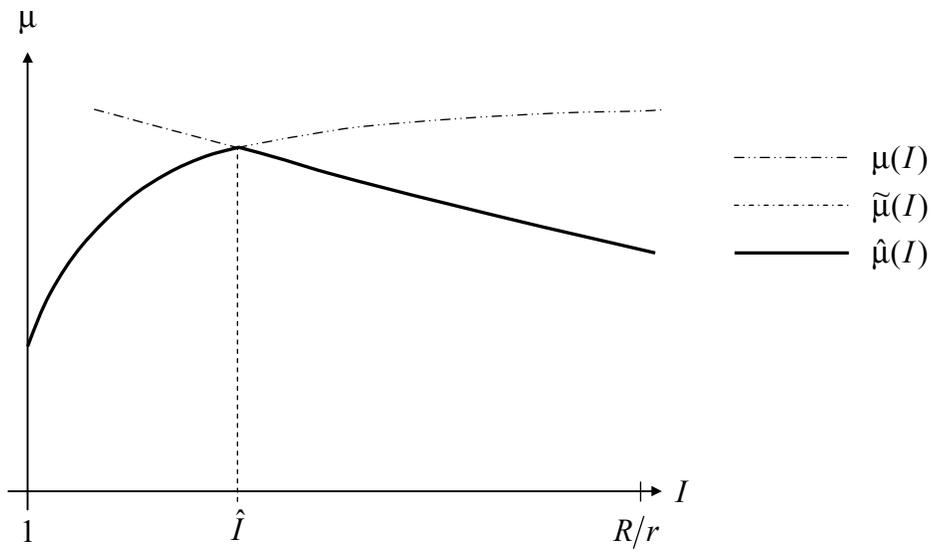
The reserve-deposit ratio and the savings rate as a function of the nominal interest rate ( $\rho < 1$ ).



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**Figure 4.**  
Equilibrium growth rate as function of the nominal interest rate.

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**Figure 5.**

## Dynamics

(a) Law of motion for  $I_t$  when  $I_t > 1$  and the incentive constraint does not bind at all dates.

(b) Law of motion for  $k_{t+1}/m_t$  when  $I_t = 1$  at all dates.

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