

Two examples of representative agent economies with monetary non-neutrality



Professor Beatrix Paal

BRB 3.114
paal@mail.utexas.edu
(512) 475-8531

Example 1: Cash-in-advance

Consider the following environment. Time is indexed by $t=0,1,2,\dots$. The economy is populated by an infinitely lived representative agent and a representative firm.

The agent is endowed with k_0 units of capital and m_0 units of fiat currency at the beginning of period 0, and one unit of time each period. She derives utility from consumption (c_t) but not from leisure. Her preferences are represented by the function

$$(1) \quad \sum_{t=0}^{\infty} \beta^t u(c_t),$$

where the function u has the usual properties. She supplies labor inelastically, owns the entire capital stock (k_t) and earns wage (w_t) and rental income ($r_t k_t$) in each period. She also purchases consumption and investment goods (i_t) each period.

The purchases of these goods are subject to the following cash-in-advance constraints. A fraction $\alpha^C \in [0,1]$ of consumption goods and a fraction $\alpha^I \in [0,1]$ of investment goods must be purchased by government supplied currency which was acquired in the previous period. Let m_t denote the number of dollars held by the consumer at the beginning of time t . Finally, the representative consumer also receives cash transfers from the government with real value τ_t always at the end of each period so that these transfers cannot be used to meet the cash-in-advance constraint.

To summarize, the representative agent maximizes (1) subject to the following sequence of budget constraints, cash-in-advance constraints and capital accumulation equations:

$$(2) \quad c_t + i_t + \frac{m_{t+1}}{p_t} \leq w_t + r_t k_t + \frac{m_t}{p_t} + \tau_t$$

$$(3) \quad \alpha^C c_t + \alpha^I i_t \leq \frac{m_t}{p_t}$$

$$(4) \quad k_{t+1} = (1 - \delta)k_t + i_t.$$

The representative firm maximizes profits taking factor prices as given. It operates the technology described by the neoclassical intensive production function

$$(5) \quad y_t = f(k_t),$$

where y_t is output per worker and k_t is capital per worker.



The government supplies fiat currency. The money stock is expanded/contracted via lump-sum transfers/taxes to the consumers. The money supply grows at the exogenously selected constant gross growth rate σ . We assume that σ is such that the cash-in-advance constraint is always binding.

Let the Lagrange multipliers associate with (2) and (3) be λ_t and μ_t . Then the first order conditions from the consumer's problem (with respect to c_t , k_t , m_t/p_{t+1}) are

$$(6) \quad u'(c_t) = \lambda_t + \alpha^C \mu_t$$

$$(7) \quad \beta \left[\lambda_t R_t + \mu_t \alpha^I (1 - \delta) \right] = \lambda_{t-1} + \mu_{t-1} \alpha^I$$

$$(8) \quad \beta \left[\lambda_t + \mu_t \right] \frac{p_{t-1}}{p_t} = \lambda_{t-1}$$

In addition to solving the consumer's problem, equilibrium values also must satisfy the firm's first order conditions,

$$(9) \quad r_t = f'(k_t),$$

$$(10) \quad w_t = f(k_t) - k_t f'(k_t),$$

and the government budget constraint:

$$(11) \quad \tau_t = \frac{m_{t+1} - m_t}{p_t}.$$

Let x^{SS} denote the steady state value of variable x . Let k^{PO} denote the steady state capital labor ratio of the economy that has no money or cash-in-advance constraint but is otherwise identical to this economy.

Combining the equilibrium conditions with the consumer's first-order condition, and evaluating the result in steady state implies the following:

$$(12) \quad u'(f(k) - \delta k) = \lambda + \alpha^C \mu$$

$$(13) \quad \beta \left[\lambda (f'(k) + 1 - \delta) + \mu \alpha^I (1 - \delta) \right] = \lambda + \mu \alpha^I$$

$$(14) \quad \beta \left[\lambda + \mu \right] \frac{1}{\pi} = \lambda$$

In (14), π is the steady state inflation rate. Note that (13) can be rewritten as

$$(15) \quad \beta \lambda f'(k) = (\lambda + \mu \alpha^I) (1 - \beta(1 - \delta))$$

OBSERVATIONS:

a) $\pi^{SS} = \sigma$.

b) (15) implies that $k^{SS} < k^{PO}$ if $\alpha^I > 0$ and $k^{SS} = k^{PO}$ if $\alpha^I = 0$.

c) When $\alpha^I > 0$,

$$\frac{dk^{SS}}{d\sigma}, \quad \frac{dy^{SS}}{d\sigma} < 0 \quad \text{and} \quad \frac{dc^{SS}}{d\sigma} < 0$$



Example 2: Reserve requirements

The economy is quite similar to the one in Example 1, except that money holdings are motivated by a reserve requirement rather than a cash-in-advance constraint. Time is indexed by $t=0,1,2,\dots$. The economy is populated by an infinitely lived representative agent, a representative firm and a representative bank.

The agent is endowed with $p_0 d_0$ dollars of bank deposits at the beginning of period 0, and one unit of time each period. She derives utility from consumption (c_t) but not from leisure so that she supplies labor inelastically. Her preferences are represented by the function

$$(1) \quad \sum_{t=0}^{\infty} \beta^t u(c_t),$$

where the function u has the usual properties.

Capital in this economy is not held directly. Consumers (are exogenously forced to) hold bank deposits as their only asset. Banks in turn hold claims on the capital stock and are also subject to a reserve requirement. At the end of period t the bank receives d_{t+1} units of goods as deposits, which it invests into k_{t+1} units of capital and m_{t+1}/p_t units of real balances. The reserve requirement is imposed on the bank's assets (rather than its liabilities): for every unit of capital investment the bank must hold θ units of real balances:

$$(2) \quad \frac{m_{t+1}}{p_t} \geq \theta k_{t+1}.$$

The bank promises to pay depositors a gross real return of ρ_{t+1} on their deposits. Each unit of capital earns a real rental income of r_{t+1} next period when rented out, and the bank also retains ownership if the undepreciated capital stock of $(1-\delta)k_{t+1}$. Fiat money does not earn nominal interest. Therefore, bank profits at $t+1$ are

$$(3) \quad k_{t+1}(1-\delta+r_{t+1}) + \frac{m_{t+1}}{p_{t+1}} - d_{t+1}\rho_{t+1}$$

The bank maximizes profits subject to the balance sheet constraint

$$(4) \quad k_{t+1} + \frac{m_{t+1}}{p_t} \leq d_{t+1}$$

and the reserve requirement (2).

Consumers take wages w_t and real return on deposits as given and maximize (1) subject to the sequence of budget constraints

$$(5) \quad c_t + d_{t+1} \leq w_t + d_t \rho_t.$$

The representative firm hires labor and rents capital and produces output according the neoclassical intensive production function

$$(6) \quad y_t = f(k_t),$$



where y_t is output per worker and k_t is capital per worker. It takes factor prices as given and maximizes profits.

The government supplies bank reserves. The money supply grows according to

$$(7) \quad m_{t+1} = \sigma m_t$$

where $\sigma > 1$ is an exogenously chosen constant. We assume that σ is large enough to imply that capital dominates reserves in rate of return. The government uses seigniorage income for purchasing government consumption.

Here, I will skip a number of steps and move directly to the analysis of the steady state. The notation is the same as in Example 1. In addition z denotes the value of real balances. ($z_t = m_{t+1}/p_t$.)

The equilibrium conditions of the economy in steady state can be summarized as follows:

$$(8) \quad \rho\beta = 1$$

$$(9) \quad \rho(1+\theta) = \frac{\theta}{\pi} + f'(k) + 1 - \delta$$

$$(10) \quad z = \theta k$$

$$(11) \quad z\left(1 - \frac{1}{\pi}\right) = g$$

OBSERVATIONS:

a) $\pi^{SS} = \sigma$.

b) $k^{SS} < k^{PO}$

c) $\frac{dk^{SS}}{d\sigma} < 0$, $\frac{dk^{SS}}{d\theta} < 0$