Stochastic Optimization of the Highland Lakes System in Texas

D. R. Krcman\(^1\); D. C. McKinney\(^2\); D. W. Watkins Jr.\(^3\); and L. S. Lasdon\(^4\)

Abstract: A multistage stochastic optimization model using linear programming was developed to provide planning tools for Lower Colorado River Authority in the operation of the Highland Lakes system, as well as a framework for examining the operation of four irrigation districts. Three primary objectives were maximized in the model: (1) Revenues from rice production, (2) recreation benefits associated with lake use, and (3) revenues from hydropower generation. The model includes stochastic inflows, weather-dependent irrigation demands, an interruptible contract decision function, a reservoir space rule to balance storage volumes between reservoirs, hydropower production, municipal and irrigation return flows, and bay and estuary inflow requirements. Model weights and coefficients were calibrated to reflect actual market prices and economic constraints, or to represent water management priorities. Stochastic optimization is used to account for meteorological uncertainty using representative inflow scenarios. Considering the uncertainty in the inflows, the model predicts optimum acreage levels, reservoir storage levels, reservoir releases, and other decisions, to maximize total expected benefits in the system.


CE Database subject headings: Optimization; Lakes; Texas; Hydrologic models; Inflow.

Introduction

Background

The Colorado River in Texas plays a crucial role in the economy and quality of life of much of central Texas. It supports a wide variety of aquatic life and waterfowl, and provides vital freshwater inflows for endangered fish and shellfish in the bay. The Colorado River also serves a variety of human purposes, including providing drinking water supplies, creating recreation opportunities, generating electricity, and irrigating crops. With increasing pressure to export water outside the basin, the influence of the Colorado River already extends beyond its watershed boundaries, and studies are underway to use the river to provide drinking water for the City of San Antonio.

Management of the Colorado River requires the joint efforts of a variety of state and federal agencies, river authorities, irrigation districts, municipalities, and other interests. Because there are demands on the river, a complex framework of water law and regulatory procedures has been established to equitably allocate the water resources. Optimization techniques to maximize benefits from water management systems have been used for several decades (e.g., Martin 1995), and have been used in this study for the Highland Lakes system, a series of six dams and reservoirs (see Table 1) operated by the Lower Colorado River Authority (LCRA) to regulate river flows on the Lower Colorado River. The two largest reservoirs in the system are Lakes Buchanan and Travis, which together contain about 2 × 106 acre ft [2,467 × 109 m3] of conservation pool storage. Both reservoirs experience variability in storage levels on an annual basis due to hydrologic fluctuations and water demands downstream. The other four reservoirs are not operated as variable storage reservoirs, and are maintained at nearly constant pool elevations. Water uses for the Highland Lakes include irrigation, municipal water supply, recreation, and hydroelectricity.

A stochastic optimization model was developed to assist LCRA in planning water delivery contracts by maximizing expected benefits accrued through water sales and recreation, while considering uncertainty in future reservoir inflows (Watkins et al. 2000). The work reported here extends that previous model to include: (1) All six Highland Lakes; (2) weather-dependent irrigation demands; (3) a water contract function; (4) a reservoir storage allocation rule for equalizing storage levels; and (5) hydropower production.

Stochastic Optimization

Because deterministic planning models are often unable to handle the uncertainties common in water resource problems (Loucks et al. 1981), stochastic optimization can be a useful tool for guiding management decisions. Instead of explicitly setting stochastic inputs at point or expected values, and ignoring uncertainty, stochastic optimization methods incorporate approximations of the probability density functions of input parameters. A wide variety of stochastic optimization methods have been applied to water resources problems.
Stochastic linear programming can handle nondeterministic parameters such as inflows when the random nature of inflows is characterized through Markov chains, chance constraints, or multiple scenario systems. To characterize inflow as a first-order Markov chain, transitional probabilities for inflow categories (e.g., high, medium, and low) are estimated based on the statistics of historical flow values (Yeh 1985). For chance constraints, the probabilities of satisfying a constraint are defined by setting random variables at determinant values according to their cumulative distribution functions (Loucks et al. 1981). For multiscenario optimization, separate streamflow hydrographs, each with a probability determined by analyzing historical streamflows, can be represented by scenario trees (Jacobs et al. 1995; Birge and Louveaux 1996; Kall and Wallace 1994; Watkins et al. 2000). The primary advantage of scenario-based stochastic programming over the other approaches is the flexibility and diversity of decision recourse it offers in modeling the decision process and defining scenarios, particularly if the state dimension is high. Typically, as done herein, the underlying probability distribution is sampled (i.e., scenarios are generated) prior to applying the solution procedure. One disadvantage is that the resulting models can be very large, requiring special solution algorithms. Previous applications of multistage stochastic programming to reservoir management (Pereira and Pinto 1985; Jacobs et al. 1995) have applied algorithms based on the L-shaped method (Benders 1962; Van Slyke and Wets 1969). This method allows the large-scale problem to be decomposed by scenario. Using this technique in a nested manner allows multistage problems to be decomposed by both scenario and decision period (Birge 1985; Gassman 1990).

### Water Management Principles for the Highland Lakes

#### Water Demands

The largest municipal water demand from the Lower Colorado River is the City of Austin. Over 150,000 acre-ft [185 x 10^6 m^3] of water were diverted by the City in 2000 (J. Musgrove, private communication, January 29, 2002); mostly from Lake Austin. The City of Austin owns water rights for more than 290,000 acre-ft [357.7 x 10^6 m^3] of Colorado River flow each year. These rights are senior to LCRA's rights to store water within the Highland Lakes, so water must be passed through the reservoir system to the diversion points without being stored when the City calls for water. Also, the City of Austin has secured additional "firm" water through contracts with LCRA; this water is stored in the Highland Lakes, and delivered at a later date, guaranteeing a total future water supply of 325,000 acre-ft [400.9 x 10^6 m^3] per year.

Four major irrigation districts rely on water from the Colorado River: Lakeside, Garwood, Pierce Ranch, and Gulf Coast Irrigation Districts. Over the period 1988–1997, the districts used a combined 417,000 acre-ft [514.3 x 10^6 m^3] per year. The Gulf Coast District is the largest water user of the four, followed by Lakeside, Garwood, and Pierce Ranch, respectively. Some relevant specifications for the districts appear in Table 2 (C. Riley, private communication, December 4, 2000; Martin 1990).

Irrigators on the Lower Colorado River divert interruptible and run-of-river flows. Interruptible water is stored in the Highland Lakes system, and then released when needed, and when available, for the irrigation districts. Since the rights to the interruptible water are owned by LCRA, these supplies may be curtailed or cut off if reservoir levels drop below predetermined minimum levels. Run-of-river flows pass through the Highland Lakes system without being stored, and are available regardless of reservoir storage levels. Run-of-river diversions are determined by separate water right and seniority policies.

The four irrigation districts are involved primarily in rice production, a water-intensive crop, and water applications are made not only to meet plant evapotranspiration needs, but also to drown competitive plants (Martin 1990). Rice is typically planted in March or early April, and harvested in July. For many rice fields, the plants are allowed to regrow after the first harvest, with second harvest between October and December. Water applications for first-crop rice typically occur from March to July, and second-crop rice requires water from August to October. Seasonal predictive models for water demand for the irrigation districts have been estimated (Nazneen 2001) for first- and second-crop seasons based on climate data such as precipita-

#### Table 1. Highland Lakes Characteristics

<table>
<thead>
<tr>
<th>Reservoir</th>
<th>Active capacity (kAF)</th>
<th>Hydro capacity (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buchanan</td>
<td>866.1</td>
<td>49</td>
</tr>
<tr>
<td>Inks</td>
<td>15.0</td>
<td>15</td>
</tr>
<tr>
<td>LBJ</td>
<td>113.7</td>
<td>54</td>
</tr>
<tr>
<td>Marble Falls</td>
<td>5.7</td>
<td>34</td>
</tr>
<tr>
<td>Travis</td>
<td>1.115.8</td>
<td>100</td>
</tr>
<tr>
<td>Austin</td>
<td>21.3</td>
<td>18</td>
</tr>
</tbody>
</table>

\(^a(1 \text{ acre-ft}=1,000 \text{acre-feet}=1,233,480 \text{m}^3).\)

#### Table 2. Irrigation District Specifications

<table>
<thead>
<tr>
<th>Area</th>
<th>Lakeside</th>
<th>Garwood</th>
<th>Pierce Ranch</th>
<th>Gulf Coast</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acres(^a) in service area</td>
<td>112,000</td>
<td>155,000</td>
<td>30,000</td>
<td>206,000</td>
</tr>
<tr>
<td>1988–1997 average</td>
<td>26,216</td>
<td>19,344</td>
<td>5,704</td>
<td>26,362</td>
</tr>
<tr>
<td>1st crop acres</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1988–1997 average</td>
<td>19,814</td>
<td>16,582</td>
<td>575</td>
<td>11,955</td>
</tr>
<tr>
<td>2nd crop acres</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>annual water use (acre-ft(^b))</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(^b(1 \text{ acre}=4,046.86 \text{m}^2=0.40468 \text{ha}).\)

\(^a(1 \text{ acre foot}=1,233.78 \text{m}^3).\)
drawal must come from stored water supplies. For firm municipal total inflow into the Highland Lakes, since any additional water right. In addition, run-of-river diversions cannot exceed the annual diversions are limited to the amount specified in the senior bound constraints. For municipal run-of-river supplies, total annual diversions and firm stored water supplies is unable to meet demands. Both run-of-river and firm water sources have upper water deficits. In practice, LCRA is obligated only to fulfill its interruptible contract with the districts. If run-of-river supplies become scarce, and the interruptible water deliveries are not enough to meet demands, LCRA suffers no direct loss as long as the interruptible contract is fulfilled. Irrigators can conceivably experience severe economic loss to their crops, whereas LCRA is not directly affected. The model uses a composite approach, in which deficits are defined as the difference between combined run-of-river and interruptible supplies and water demands. Penalties can therefore be incurred even when LCRA's contract is met. LCRA assigns shortages to irrigation districts in proportion to a district's interruptible stored water demand.

**Hydropower**

A subsidiary consideration in Highland Lakes operation is hydropower generation. Thirteen turbines are used to produce electricity from hydropower releases, with at least one turbine installed in each of the six dams. The units have a combined capacity of about 270 MW, and are the cheapest energy source for LCRA energy customers. Similar to recreation demands, hydropower generation is subordinate to other uses of the reservoir system, and can only be optimized under constraints that protect competing water interests (LCRA 1999). Hydropower generation is a nonlinear function of the flow discharged through a penstock and the elevation difference between pool and tailwater levels. Hydropower generation is represented reasonably well as a function of flow in this system because head does not vary widely. Flow rates vary over a much greater range, in contrast, even on a monthly basis. Monthly average pool and tailwater elevations for 1942–2001 were used, along with values of generated power under varying flows and elevation heads to develop linear relations between hydropower generation and monthly releases. Similar linearization techniques have been used in studies of hydropower generation in the Highland Lakes in the past, and have yielded satisfactory results (Martin 1995). The parameters for hydropower of the six Highland Lake dams are shown in Table 3.

**Reservoir Balancing**

To equitably distribute storage and releases between the two variable storage reservoirs, Lakes Buchanan and Travis, LCRA has developed guidelines for releases from the reservoirs over the course of the year (Q. W. Martin, private communication, Febru-

![Graph](image)

**Fig. 1.** Interruptible storage contract decision function (1 acre ft = 1,233.48 m³)

**Interruptible Water Contracts**

LCRA has developed a piecewise linear decision function to determine the maximum amount of stored water to be contracted for release to irrigators over the growing season based on the combined storage of Lakes Buchanan and Travis on January 1 of any year (see Fig. 1 for the decision function used in this model). Using this function, LCRA enters into water delivery contracts with the irrigation districts, thereby providing farmers with some indication of available water supplies for the upcoming growing season. When available water supplies cannot meet irrigation districts demands, LCRA allocates water (LCRA 1999) so that all irrigation districts incur deficits in proportion to their demand for stored water.

Although LCRA attempts to accommodate the recreation industry on all the Highland Lakes, municipal and irrigation water uses have higher priority (LCRA 1999). Interruptible water sales for purposes outside of the irrigation districts can be restricted or terminated if the lakes fall to levels adverse to recreation interests, but agricultural and municipal diversions will continue to be made regardless of impacts on recreation. Studies have been done on the direct economic importance of recreation on the Highland Lakes (USACE 1992, 1994). By using estimates derived from these studies, linear approximations for monthly recreation economic benefits as functions of reservoir storage have been developed (Watkins et al. 2000).

**Water Deficits and Penalties**

If water demand for either the irrigation districts or the City of Austin is unmet, penalties apply to the model’s objective function to represent the loss in economic benefits. For municipal water diversions, deficits occur when the combination of run-of-river diversions and firm stored water supplies is unable to meet demands. Both run-of-river and firm water sources have upper bound constraints. For municipal run-of-river supplies, total annual diversions are limited to the amount specified in the senior water right. In addition, run-of-river diversions cannot exceed the total inflow into the Highland Lakes, since any additional withdrawal must come from stored water supplies. For firm municipal water, the sum of annual stored and run-of-river withdrawals must be less than the contract amount established with LCRA.

When irrigation district demands exceed available water from run-of-river and stored interruptible supplies, the districts incur water deficits. In practice, LCRA is obligated only to fulfill its interruptible contract with the districts. If run-of-river supplies become scarce, and the interruptible water deliveries are not enough to meet demands, LCRA suffers no direct loss as long as the interruptible contract is fulfilled. Irrigators can conceivably experience severe economic loss to their crops, whereas LCRA is not directly affected. The model uses a composite approach, in which deficits are defined as the difference between combined run-of-river and interruptible supplies and water demands. Penalties can therefore be incurred even when LCRA's contract is met. LCRA assigns shortages to irrigation districts in proportion to a district's interruptible stored water demand.

**Hydropower**

A subsidiary consideration in Highland Lakes operation is hydropower generation. Thirteen turbines are used to produce electricity from hydropower releases, with at least one turbine installed in each of the six dams. The units have a combined capacity of about 270 MW, and are the cheapest energy source for LCRA energy customers. Similar to recreation demands, hydropower generation is subordinate to other uses of the reservoir system, and can only be optimized under constraints that protect competing water interests (LCRA 1999). Hydropower generation is a nonlinear function of the flow discharged through a penstock and the elevation difference between pool and tailwater levels. Hydropower generation is represented reasonably well as a function of flow in this system because head does not vary widely. Flow rates vary over a much greater range, in contrast, even on a monthly basis. Monthly average pool and tailwater elevations for 1942–2001 were used, along with values of generated power under varying flows and elevation heads to develop linear relations between hydropower generation and monthly releases. Similar linearization techniques have been used in studies of hydropower generation in the Highland Lakes in the past, and have yielded satisfactory results (Martin 1995). The parameters for hydropower of the six Highland Lake dams are shown in Table 3.

**Reservoir Balancing**

To equitably distribute storage and releases between the two variable storage reservoirs, Lakes Buchanan and Travis, LCRA has developed guidelines for releases from the reservoirs over the course of the year (Q. W. Martin, private communication, Febru-

**Table 3.** Hydropower Parameters for the Highland Lakes

<table>
<thead>
<tr>
<th>Reservoir</th>
<th>Head (ft)</th>
<th>Regression slope (MW·d/kacre-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buchanan</td>
<td>125</td>
<td>4.58</td>
</tr>
<tr>
<td>Inks</td>
<td>60</td>
<td>1.95</td>
</tr>
<tr>
<td>LBJ</td>
<td>83</td>
<td>2.97</td>
</tr>
<tr>
<td>Marble Falls</td>
<td>54</td>
<td>1.77</td>
</tr>
<tr>
<td>Travis</td>
<td>175</td>
<td>6.50</td>
</tr>
<tr>
<td>Austin</td>
<td>61</td>
<td>2.19</td>
</tr>
</tbody>
</table>

1 ft = 0.3048 m.

2 MW·d/kacre-ft = 68,586 J/m³.
ary 12, 2002). From October to March, storage levels are adjusted so the fraction of used capacity in each reservoir is roughly equal. In April, diversions of irrigation water begin, and calls may be made for releases from the Highland Lakes. From April to July, releases are made from Lake Travis, in large part to create additional flood storage capacity in this downstream reservoir. From July to August, irrigation releases are made from upstream Lake Buchanan.

Model Formulation

The time horizon chosen for the Highland Lakes model is five years, long enough to simulate a persistent drought in the system and to prevent “end effects” from influencing first-stage decisions in the model. A monthly time step is used, with January as the initial month.

The model is a simplified representation of the six reservoirs; one main inflow from the Colorado River upstream of Lake Buchanan and two tributary inflows from the Llano and Pedernales Rivers into Lakes LBJ and Travis, respectively; one municipal diversion from Lake Austin and one return flow from the City of Austin wastewater discharges; and diversions to four irrigation districts and return flow from irrigation runoff (see Fig. 2). Flows not diverted, or those that are reintroduced as return flows, continue downstream to Matagorda Bay.

The model’s objective function includes several weighted terms, with the main items being revenue from crop production, recreation benefits, hydropower generation, irrigation water deficits and municipal water deficits. Other minor factors, described later, are also included in the objective function.

Scenario-Based Optimization

Scenario-based stochastic optimization is used to model decisions accounting for the uncertainty in future inflows into the Highland Lakes. In the model, stages are defined as times when major decisions are made that affect water management in the basin, e.g., the beginning of each year when the area planted to rice is decided. A scenario tree, Fig. 3, illustrates how the stochastic variables evolve in the model; the root of the tree corresponds to the immediately observable, deterministic data and the first-stage (“here and now”) decisions. The nodes of the tree at subsequent stages correspond to times when realizations of stochastic processes diverge and new decisions must be made. These decisions depend on the realizations of the random process up to that time, on previous decisions, the likelihood of future outcomes and the availability of future recourse decisions.

The model is designed primarily to optimize the first-stage decisions related to the interruptible water contract and rice planting that must be made immediately. Future decisions, such as reservoir operations, represent the multistage decision process faced by water managers, and, to some extent, are “simulated.” In the first year, five different possible hydrographs of equal probability are represented by the five branches emanating from the first node of the diagram (Stage 1). These branches represent very dry, dry, average, wet, and very wet inflow hydrographs. For year two, three hydrographs originate from the ends of each of the first-year branches. These are also equally probable hydrological scenarios, and together define 15 different streamflow hydrographs up to the end of year two (although the first six scenarios have the same first year flows, the first two scenarios have the same second year flows, etc.). At the end of year two, the scenario tree branches again, dividing the second-year branches into two new branches, each of which defines an equally probable three-year hydrograph for years three through five. The result is 30 different streamflow hydrographs or scenarios. The stochasticity represented in the model is simply for “year types,” with the purpose of the model being to support the first-stage annual contract decision. A monthly time step is used to represent seasonal variations in, e.g., hydrologic processes, environmental requirements and reservoir operations in a more realistic manner.

One benefit of using a scenario tree is that the decision process is designed so current decisions do not depend on specific future events. Nonanticipativity constraints in the model impose the requirement that decisions up to a certain point must coincide for those scenarios which have common root-to-leaf paths up to that level in the scenario tree. In the scenario tree represented in Fig. 3, scenarios one through six all share identical inflow hydrographs for the first year. Since all six scenarios start with the same initial storage conditions, decisions for these six scenarios should be identical up to the end of the first year. For example, in scenarios one through six, the area of rice planted (the major decision variable affecting interruptible water demand) at the beginning of the first year and the reservoir releases throughout the year constrained to be the same for all six scenarios. In fact, all thirty scenarios share the same first year planting decision, since this decision must maximize performance for all future scenarios from common initial conditions.
Scenarios for the model were generated using a quantile sampling technique with the historical hydrographs for the period 1941–1965 (Watkins et al. 2000) because this period includes the 1947–1956 drought of record and it is consistent with records of available inflows (i.e., with senior water rights subtracted out) as determined for a drought management plan (LCRA 1999). First, the annual hydrographs for the three tributaries (Colorado, Pedernales, and Llano Rivers) were temporally linked to preserve spatial correlation. Next, the annual hydrographs were ranked in terms of total cumulative flow, and divided into quantile groups. Annual hydrographs for the linked inflows were picked at random from particular percentile groups according to the scenario tree (Fig. 3). For instance, for the first year, groups were formed for each of the five 20% quantiles, and one-year hydrographs were selected randomly from each of the quantile groups and assigned to branches in the tree. A similar sampling procedure applied to the second year, where three quantile groups were used. For years three through five, two quantile groups were formed using three-year hydrographs to partially preserve autocorrelation.

While this scenario generation technique preserves spatial correlation between the three tributaries, it also has disadvantages, including a lack of strong year-to-year autocorrelation. Interannual autocorrelation of streamflow in this system is small, and not significantly different from zero at the 0.05 significance level. However, some authors have shown that it can have a significant impact on expected storage levels (Vaugh and Maidment 1987). Our method of constructing the scenario tree partially accounts for the (small) interannual correlation by considering three-year “blocks” in the last stage. Watkins et al. (2000) describe a sampling technique based on the nearest-neighbor bootstrap method (Lall and Sharma 1996) that better preserves the annual correlation of flows.

**Objective Function**

The objective function can be represented by the following expression:

Maximize $Z = CRPTOT + RECTOT + HYDTOT + STOTOT - DEFTOT - MISCTOT$  

The components of the objective function are described as follows.

- Total revenue from crop production is a linear function of the first- and second-crop planted areas

$$CRPTOT = \frac{12}{ST} \sum_{tc} \sum_{s} \sum_{id} c_{t1} ACR_{tc,s,ida}$$

where $S$=total number of scenarios; $T$=number of time periods; $s$=individual scenario; $tc$=January month subset of $t$, the set of monthly time periods; $id$=set of irrigation districts; and $c_{t1}$=crop period.

The coefficients $c_{t1}$ and $c_{t2} (\$/acre) represent unit profit from rice production for the first and second crops (Rister et al. 1989; Schultz 1996). Monthly benefits for both crops, all districts, and all scenarios are summed to yield an expected annual value.

- Recreation benefits are estimated for Lakes Buchanan and Travis

$$RECTOT = \frac{12}{ST} \omega_{l} \sum_{rrs} \sum_{t} \sum_{s} REC_{rrs,t,s}$$

where $rrs$=set of recreation reservoirs and $\omega_{l}=0.1$ (resulting in a balance between maintaining storage and meeting downstream water demands). Recreation benefits for each month are calculated as a linear function of average reservoir storage for each month for two separate seasons, “Winter” (September–May) and “Summer” (June–August) (Watkins et al. 2000).

- Hydropower generation revenues are approximated as a linear function of reservoir releases

$$HYDTOT = \frac{12}{ST} \omega_{r} \sum_{rrs} \sum_{t} \sum_{s} 24,000mp_{rs} RHYDRO_{rrs,t,s}$$

where $rs$=set of reservoirs and $\omega_{r}=5$ cents/kWh, which is close to current utility rates.

- Expected reservoir storage benefit in the last time step is calculated as

$$STOTOT = \frac{1}{S} \sum_{rrs} \sum_{s} \omega_{r3,rs} SR_{rrs,s,60,s}$$

where $rs$=set of storage reservoirs and $\omega_{r3,rs}$ have values of $20$ and $17$ per acre-ft for Lakes Buchanan and Travis, respectively. This minimizes end-of-horizon effects by preventing the reservoirs from draining during the last few months.

- Lost revenue from municipal and irrigation water deficits is included as

$$DEFTOT = \frac{12}{ST} \left[ \left( \sum_{tc} \sum_{s} cmUM_{t,s} \right) + \left( \sum_{id} \sum_{s} \sum_{c=1,2} cu_{t} UI_{id,c,s} \right) \right]$$

with $cm=$$600/acre-ft ($0.49/m^3$) as the marginal loss from municipal deficits and $cu_{1} =$ $120/acre-ft ($0.10/m^3$) and $cu_{2} =$ $100/acre-ft ($0.08/m^3) as the marginal losses due to irrigation deficits, respectively.

- Slack, surplus, and secondary variables including: (1) slack and surplus variables for the interruptible water contract; (2) elastic variables for proportional irrigation deficits; (3) equity constraints for planted acreages; (4) reservoir releases above recommended levels; (5) penalty for deviations from equitable storage between Lakes Buchanan and Travis; (6) penalty for municipal firm water withdrawals; and (7) penalty to ensure that run-of-river supplies are used up before interruptible supplies.

**Model Constraints**

Constraints for the Highland Lakes model are described below, along with the corresponding equations where practical. A complete list of all equations, parameters and data used in the model can be found in Kracman et al. (2002).

- Mass balance for each reservoir.
- Minimum storage levels for Lakes Buchanan and Travis.
- Balancing rule to equalize the fractional storages of Lakes Buchanan and Travis.
- Linear interruptible contract function
\[ Y_{tc,s} = YB + YM(SR_{tBu,s} + SR_{tTrv,s}) + ELVA_{tc,s} - ELVB_{tc,s}, \quad \forall t,s \]  

(7)

where \( Y_{tc,s} \) = annual interruptible water contract (kAF/year); \( YM \) and \( YB \) = interruptible contract line segment slope and intercept, respectively; \( SR_{tBu,s} \) = storage in lake \( rs \) at the end of period \( t \), scenario \( s \); and \( ELVA_{tc,s} \) and \( ELVB_{tc,s} \) = elastic variables penalizing interruptible contract deviations.

- Separation of reservoir turbine and spill flows.
- Separation of reservoir releases into “normal” and “excess” flows.
- Upper limit is used for annual municipal run-of-river diversions.
- City of Austin deficit defined as the difference between monthly municipal demands and the sum of firm and run-of-river diversions.
- Municipal return flows (estimated at 55% of the municipal diversions).
- Matagorda Bay inflows defined as flow past Bastrop minus the consumptive fraction of interruptible irrigation diversions, plus run-of-river irrigation diversion return flows (30%).
- First- and second-crop water demands.
- Seasonal irrigation demands decomposed into monthly demands.
- Irrigation deficits defined as the difference between monthly crop demand and the sum of run-of-river and interruptible diversions.
- Inequalities between irrigation district diversions.
- Total interruptible water diversions for each district limited to contract amount

\[ \sum_{t \in tc} \sum_{id} \text{INTR}_{id,t,s} \leq Y_{tc,s}, \quad \forall s \]  

(8)

where \( \text{INTR}_{id,t,s} \) = interruptible diversion for district \( id \) in period \( t \), scenario \( s \).

- Second-crop rice areas limited to less than first-crop areas.
- Deviations from historical (1988–1997) first- and second-crop planted areas (\( c = 1,2 \)).
- Seasonal run-of-river withdrawals calculated from monthly withdrawals for the irrigation districts in both the first- and second-crop periods.
- Interruptible irrigation water withdrawals for each first- and second-crop period (\( c = 1,2 \)).
- Nonanticipativity constraints for decisions made under the same observed hydrologic conditions, including reservoir releases, interruptible storage contracts, and first- and second-crop planting decisions.
- Additional constraints include: capacity constraints for each reservoir; seasonal upper bounds on recreation benefits; interruptible contract bounds; limits on run-of-river diversions; bay and estuary flow requirements; and bounds on first-crop acreage.

**Solution Procedure**

The objective function and constraints comprise a multistage, stochastic LP model consisting of over 130,000 columns, 82,000 rows, and 334,000 nonzeros. Solution of problems of this scale requires complex algorithmic procedures, for which there are several computer packages available. The General Algebraic Modeling System was chosen as the modeling platform, since it was used in previous work (Watkins et al. 2000), and because of its flexibility and ease of use. The CPLEX solver was used, because of its speed. Solution times for a Pentium III processor were about one CPU minute, and required a little over 37,000 simplex iterations. CPLEX presolve and infeasibility analysis resulted in a reduced linear program with about 44,600 rows, 62,800 columns, and 153,500 nonzeros.

Several components of the model have been represented in linear form (hydropower generation, recreational benefits) to keep the model computationally tractable. These result in a somewhat simplified model. In addition, crop production has not been represented in the model, thus neglecting the effects of diminishing marginal returns from agricultural production. The tightly constrained nature of the model makes it somewhat of a mix of simulation and optimization.

**Results**

Results of the model were analyzed for a variety of different purposes, including the analysis of crop production and water use in the irrigation districts, and prediction of future reservoir storage levels. Sensitivity analysis was conducted to assess the relative importance of individual components of the model.

**Reservoir Storage Levels**

One useful model output is reservoir storage volumes as a function of time. Fig. 4 shows predicted 10th, 50th (median), and 90th percentiles for Lake Buchanan. (Lake Travis levels were similar except for a higher level corresponding to the larger conservation pool volume.) Median storage levels for both lakes are significantly below historical averages, because the scenarios have lower minimum flows (sometimes zero) than the historic flow. For all but the 90th percentile plot, there is also a noticeable decreasing trend in storage for both reservoirs because there are benefits to emptying the reservoirs over the five-year time horizon.

Variations in storage levels can also be viewed using discrete cumulative distribution plots at specified times. Fig. 5 shows the cumulative distribution for Lake Buchanan on July 1, Year 1. The five vertical lines in the distribution reveal the effects of the tree structure and nonanticipativity constraints, which require releases within the five groups of six scenarios to be equal for the first year. The spread of the distribution is fairly significant, ranging...
from a little over 400 to about 830 kacre-ft, just below capacity, and there is a sharp differentiation between the drier three scenario groups and the wetter two, where storage values jump from 550 to about 800 kacre-ft. The cumulative distribution for Lake Travis storage is similar to that for Lake Buchanan in terms of its overall shape due to the influence of the space rule, but it is shifted out to higher storage volumes because of its larger capacity. Moving six months into the future and considering the cumulative distribution for Lake Buchanan at January 1, Year 2, the range of possible storage levels drops dramatically, covering a span from about 350 to 480 kacre-ft. As in the earlier time period, the distribution for Lake Travis is similar to that for Buchanan, except it is shifted to higher storage values. While the model output shows deviations from the storage allocation rule from time to time, relative storage levels stay fairly constant between Lakes Buchanan and Travis within scenario groups. These results help water resource planners to anticipate reservoir levels that are likely to result from the decisions prescribed by the model.

Planted Acreage Levels

Planted acreage levels for both first- and second-crop rice are critical variables in the Highland Lakes model and the actual river system. A graph of the 10th, 50th, and 90th percentiles for combined irrigated acreage in all districts is shown for the first and second crops in Fig. 6. First-crop rice for Year 1, constrained by nonanticipativity to be the across all scenarios, is predicted to be planted on about 79,000 acres (31,970.2 ha), compared to an average of 77,600 acres (31,403.6 ha) planted over 1988–1997. Variability increases in Years 2–4, with median first-crop acreage levels dropping to 44,000 acres (17,806.2 ha) in Year 2 and about 25,000 acres (10,117.1 ha) for Year 4. The small median acreage levels for these years result from some low-flow scenarios and possibly end effects in the model.

In Year 5, all three percentiles correspond to acreage levels of about 25,000 acres (10,117.1 ha). The reason for the convergence of percentiles in Year 5 is related to the final storage benefit equations used to reduce end-of-horizon effects on reservoir storage levels. Since the marginal benefits assigned to final period storage are uniform across all scenarios, this has an impact on the marginal value of first-crop rice acreage in Year 5 as well.

Interruptible Storage Contracts

Interruptible contracts are highly constrained in the model to follow LCRA’s contract decision function. While contract levels are largely dependent on storage volumes, the actual diversions made for irrigation use are less constrained. Fig. 7 shows predicted contracts, interruptible diversions, run-of-river diversions, and total diversions for all five years. These values were obtained by calculating mean values for each year considering all 30 scenarios. Contract amounts remain fairly constant for all five years, falling only slightly from 272 kacre-ft (335.5 million m³) in Year 1 to 255 kacre-ft (277.5 million m³) in Year 5. Interruptible diversions are below the contract amount, and become a smaller fraction of the associated contract amount over time. Even though storage levels are high enough to allow for full interruptible contracts, districts sometimes choose not to withdraw their full contract.

The reasons for irrigators not withdrawing their full contract are related to very low flow conditions for some scenarios. Even if storage in Year 1 is sufficient to meet the full interruptible contract, irrigators may be hedging against the future, when in-

Fig. 5. Lake Buchanan cumulative distribution of storage (July 1, Year 1) (1 kacre-ft=1,000 acre-ft=1,233,480 m³)

Fig. 6. Predicted first-crop acreage percentiles (1 acre =4,046.86 m²=0.40468 ha)

Fig. 7. Predicted interruptible contracts and irrigation diversions (1 kacre-ft=1,000 acre-ft=1,233,480 m³)
flows might drop dramatically. Since Fig. 7 shows average (mean) results across all scenarios, very dry scenarios may have a significant impact on the expected values shown in the graph.

Likewise, run-of-river diversions also show a decreasing trend over time, starting at about 224 kacre-ft (276.3 million m³) in Year 1, just short of the maximum annual supply of about 230 kacre-ft (283.7 million m³), to 180 kacre-ft (222.0 million m³) in Year 5. Since available run-of-river supplies for the model were set equal across scenarios, and repeat each year, it seems counterintuitive for the districts to not withdraw all run-of-river supplies every year. Run-of-river water is assumed, for the purposes of this model, to come from a source external to the river. Consumptive use of the run-of-river supply would have no negative effects on other water demands, and return flows from run-of-river use would help meet the bay and estuary flow requirements. Acreage levels and water deficits are, however, linked between the irrigation districts through the equity constraints. These constraints result in acreage levels being limited in some districts, such that the crops in those districts do not require the full run-of-river supply for all months. Furthermore, available run-of-river supplies are indexed by irrigation district, so it is possible for one district to use all its run-of-river supplies while being unable to access unused supplies from another district. If the acreage equity constraints prevented an underutilizing district from boosting its crop base to make use of the available run-of-river water, unused supplies would result.

Conclusions

A multistage stochastic optimization model using linear programming was developed, building on earlier research (Watkins et al. 2000), to provide planning tools for LCRA in the operation of the Highland Lakes system. A framework was also created for examining the operation of four irrigation districts which make up the largest water demand on the Lower Colorado River. Three primary objectives were maximized through a weighted objective function in the model: (1) revenues from rice production, (2) recreation benefits associated with lake use, and (3) revenues from hydropower generation. Secondary objectives included minimizing municipal and irrigation water deficits, maximizing benefits of final storage in order to reduce end-of-horizon effects, and maintaining equity among the irrigation districts. Capabilities added to the new model include the use of weather-dependent irrigation demands, a linear approximation of LCRA’s interruptible contract decision function, a modified space rule for Lakes Buchanan and Travis, a linear approximation for calculating hydropower production, inclusion of municipal and irrigation return flows, minimum bay and estuary flow requirements, and other features. The new capabilities allow for a more accurate representation of the Colorado River system, and should help produce more reliable results. Model weights and coefficients were calibrated to reflect actual market prices and economic constraints, or to represent water management priorities. Results showed that there is a decreasing trend in storage due to benefits from emptying the reservoirs over the five-year time horizon.

In the model, reservoir operations are optimized within the interruptible contract function to maximize total benefits, but the highly constrained nature of the model results in a somewhat simulated result. Stochastic optimization is able to account for meteorological uncertainty by considering the probabilistic structure of events, and using scenarios that preserve the moments of the observed probability distributions. By using the operating policy of the interruptible contract function, and assigning benefits to other water uses, the model predicts optimum acreage levels, reservoir storage levels, reservoir releases, and other decisions, to maximize total benefits in the system. This information can be used to evaluate possible future conditions, while also suggesting appropriate immediate actions.

Discrete cumulative distributions for reservoir storage volumes, providing statistical inferences as to future operating levels, are computed by the model. Understanding the probability of retaining a certain amount of storage at a given time in the future is extremely beneficial for ensuring the availability of municipal and irrigation water supplies. The model also prescribes optimal acreage levels under uncertainty, and predicted acreage levels provide useful information as to how best to distribute limited irrigation supplies and minimize crop water deficits. With municipal demands for Colorado River water expected to continue to rise, and with LCRA owning all water rights for the four irrigation districts, the acreage levels prescribed by the model could provide objective suggestions if difficult choices must be made in the future as to allowable rice acreage levels.

Acknowledgments

In kind memory of Dr. Quentin W. Martin who led by example in the field of water resource systems analysis. His continual interest and encouragement in the type of work reported here were always motivating for his colleagues. Partial funding for this research was provided by the Lower Colorado River Authority (LCRA). The detailed, thoughtful and helpful comments of several reviewers are appreciated.

References


