Derivable rules for ¬, &

Indirect proof (second form)

n. Show ~~Φ~~ Φ

n+1. ¬Φ AIP

p. Ψ

q. ¬Ψ

¬(¬P&Q), Q ∴ P s1.05

Commutativity of Conjunction (&C)

n. (Φ & Ψ)

m. (Ψ & Φ) &C, n

Associativity of Conjunction (&A)

n.	$((\Phi \,\&\, \Psi) \,\&\, \theta)$	
m.	$(\Phi \,\&\, (\Psi \,\&\, \theta))$	&A, n

Rules for the → and the ↔

Conditional Exploitation

n.	$(\Phi \rightarrow \Psi)$	
m.	Φ	
p.	Ψ	→E, n, m

Conditional Proof

n.	~~Show~~ $(\Phi \rightarrow \Psi)$	
n+1.	Φ	ACP
n+ p.	Ψ	

(¬P&Q) ∴ (P→R) sl.06

(A&¬B) ∴ ¬(A→B) sl.061

Biconditional Introduction

n.	$\Phi \rightarrow \Psi$	
m.	$\Psi \rightarrow \Phi$	
p.	$\Phi \leftrightarrow \Psi$	↔I, n, m

Biconditional Exploitation

n.	$(\Phi \leftrightarrow \Psi)$		
m.	Φ	(or Ψ)	
p.	Ψ	(or Φ)	↔E, n, m

(P↔Q), (P↔R) ∴ (Q↔R) sl.07

Derivable rules for →, ↔

Conditional Exploitation* (→E*)

n.	(Φ → Ψ)	
m.	¬Ψ	
p.	¬Φ	→E*, n, m

Biconditional Exploitation* (↔E*)

n.	(Φ ↔ Ψ)		
m.	¬Φ	(or ¬Ψ)	
p.	¬Ψ	(or ¬Φ)	↔E*, n, m

Negation-Conditional (¬ →)

n.	¬(Φ → Ψ)	
m.	(Φ & ¬Ψ)	¬→, n

Negation-Biconditional (¬↔)

n. $\underline{\neg(\Phi \leftrightarrow \Psi)}$

m. $(\neg\Phi \leftrightarrow \Psi)$ (or $\Phi \leftrightarrow \neg\Psi$) ¬↔, n

$\neg(P \rightarrow Q)$ ∴ $(P \& \neg Q)$ sl.08

Rules for $\vee$

Disjunction Introduction ($\vee$I)

n.	Φ (or Ψ)	
n + p.	$(\Phi \vee \Psi)$	$\vee$I, n

Disjunction Exploitation ($\vee$E)

n.	$(\Phi \vee \Psi)$	
m.	$(\Phi \rightarrow \theta)$	
p.	$(\Psi \rightarrow \theta)$	
q.	θ	$\vee$E, n, m, p

$(\neg P \vee \neg Q) \therefore \neg(P \& Q)$ sl.09

Derivable rule for $\vee$

De Morgan's Laws

Negation-Conjunction

n.	$\underline{\underline{\neg(\Phi \,\&\, \Psi)}}$	
m.	$\neg\Phi \vee \neg\Psi$	$\neg\&$, n

Negation-Disjunction

n.	$\underline{\underline{\neg(\Phi \vee \Psi)}}$	
m.	$\neg\Phi \,\&\, \neg\Psi$	$\neg\vee$, n

$(P \vee Q) \therefore (Q \vee P)$ sl.10

Conditional-Disjunction

n.	$\underline{\underline{(\Phi \rightarrow \Psi)}}$
m.	$(\neg\Phi \vee \Psi)$

Commutativity of Disjunction

n.	$\underline{\underline{(\Phi \vee \Psi)}}$	
m.	$(\Psi \vee \Phi)$	$\vee$C, n

Associativity of Disjunction

n.	$((\Phi \vee \Psi) \vee \theta)$	
m.	$(\Phi \vee (\Psi \vee \theta))$	$\vee$A, n

Disjunction Exploitation* ($\vee$E*, MTP)

n.	$(\Phi \vee \Psi)$		
m.	$\neg\Phi$	(or $\neg\Psi$)	
p.	Ψ	(or Φ)	$\vee$E*, n, m

Proof Strategies

To get:	Try:
P, Q, R ,...	Indirect Proof
¬Φ	Indirect Proo f
(Φ & Ψ)	Prove Φ and Ψ separately, use Conjunction Intro
(Φ ∨ Ψ)	Use indirect proof, ¬∨
(¬Φ ∨ Ψ)	Use conditional-disjunction
(Φ → Ψ)	Use Conditional Proof.
(Φ ↔ Ψ)	Prove Φ → Ψ and Ψ → Φ. separately, use ↔ Intro.

To exploit:	Try:
¬Φ	Use →E*, ∨E*, ¬¬, ¬&, ¬→, ¬↔, or ¬∨.
(Φ & Ψ)	Use &E to get Φ and Ψ.
(Φ ∨ Ψ)	(a) Prove ¬Φ or ¬Ψ, and use ∨E*, or (b) Prove Φ → ξ and Ψ → ξ, and use ∨E,
(Φ → Ψ)	Prove Φ or ¬Ψ and use →E or →E*.
(Φ ↔ Ψ)	Prove Φ, ¬Φ, Ψ, or ¬Ψ and use ↔E or ↔E*.

1. When in doubt, use indirect proof.
2. If you've started an indirect proof and don't know what to do next, pick a sentence letter and try to prove it by another indirect proof.

$\neg(\neg P \& Q)$, $((P \vee R) \rightarrow \neg(Q' \& S))$, $(Q \leftrightarrow S)$ $\therefore$ $(Q' \rightarrow \neg(Q \vee S))$ sl.11

$\therefore$ $(((P \& Q) \rightarrow R) \leftrightarrow ((P \& \neg R) \rightarrow \neg Q))$ sl.12

$\neg(P \rightarrow \neg Q)$, $(R \rightarrow (\neg P \vee \neg Q))$, $((R \vee S) \leftrightarrow S')$ $\therefore$ $(S' \leftrightarrow S)$ sl.13

$\therefore$ $((P \rightarrow (Q \vee R)) \leftrightarrow ((P \rightarrow Q) \vee (P \rightarrow R)))$ sl.14

$(S' \rightarrow S)$, $\neg(P \& R)$, $(Q \vee (R \rightarrow S'))$ $\therefore$ $((P \vee \neg Q) \rightarrow (\neg R \vee S))$ sl.15