Phl 313K
Profs. Asher/Koons
Fall 1996

SECOND MAJOR EXAM

I. Derivations. Use your blue book. You may use derivable rules.

A. 22 points: $\forall x \forall z(\exists y(Hxy \& Hyz) \rightarrow Jxz)$, $\neg\forall x \neg Hxx \vdash \exists y Jyy$

B. 25 points. $\forall x\ f(x) = g(g(x))$, $\vdash \forall x\ f(g(x)) = g(f(x))$

PART II MULTIPLE CHOICE

4 points each. Choose the best answer and fill in the corresponding circle on your answer grid.

A. Syntax

1. Which of the following is not a well-formed formula (by the standards of *Deduction*)?

(A) $\forall x \exists y(Fxy \leftrightarrow (Gyx \& Gxy))$
(B) $\forall x \forall y(Fx \& (Gyx \rightarrow Gxy))$
(C) $\forall x(\forall y(Gy \rightarrow Fx) \rightarrow Hxy)$
(D) $(\forall x Fx \vee \exists y(Fyy \rightarrow Gy))$
(E) $\exists y(Fy \rightarrow \forall x(Fxy \rightarrow Hyx))$

B. Translation: For questions 1-5, select the best translation of the English sentence into the symbols of formal logic. [Some of these sentences may be obviously false.] Use the following scheme of abbreviation:

$\varnothing$: the empty set
$S\alpha$: α is a set.
$T\alpha\beta$: α is a subset of β
$E\alpha\beta$: α is a member of β

2. Every member of a is a subset of some member of b.

(a) $\exists x(Exa \& Tx\exists y\ Eyb)$
(b) $\forall x(Exa \rightarrow \exists y\ (Eyb \& Exb))$
(c) $\forall x(Exa \rightarrow \exists y(Eyb \& Txy)$
(d) $\forall x(Exa \rightarrow \exists y(Eyb \& Tyx))$
(e) $\forall x(Txa \rightarrow \exists y(Eyb \& Exy)$

3. A has at most one member.

(a) $\forall x(Exa \rightarrow x = x)$
(b) $\forall x \exists y(Exa \rightarrow x = y)$
(c) $\forall x \forall y(x = y \rightarrow Exa)$
(d) $\forall x \forall y(Exa \& Eya \rightarrow x = y)$
(e) $\exists x \forall y(Exa \& (Eya \rightarrow x = y))$

4. The empty set is the only thing which is a subset of every set.
(a) $\forall x \forall y((Sy \& Txy) \rightarrow x = \varnothing)$
(b) $\forall x(\forall y(Sy \rightarrow Txy) \rightarrow x = \varnothing)$
(c) $\forall x(x = \varnothing \rightarrow \forall y(Sy \rightarrow Txy))$
(d) $\forall x(Sx \rightarrow T\varnothing x)$
(e) $\forall x \exists y((Sy \& Tyx) \rightarrow x = \varnothing)$

B. Structures or QL Models.

5. Consider $\exists zFz \rightarrow \forall z Fz$
(A) The formula is true in every structure and so a valid formula of QL.
(B) The formula is true in every structure but not a valid formula of QL.
(C) The formula is not true in every structure but it is satisfiable.
(D) The formula is false in every structure and so a contradictory formula of QL

6. Which of the following is not a valid formula of QL.
(A) $\forall y \exists x\, f(y) = x$
(B) $\forall x \forall y(x = y \rightarrow f(x) = f(y))$
(C) $\forall x \forall y(f(x) = f(y) \rightarrow x = y)$
(D) $\forall x(Fx \rightarrow Fx)$

7. Consider the collection of models in which $\forall x(x = a \vee x = b)$ is true.
In that class of models the formula $\forall x Ff(x)$ is equivalent to the (quantifier free) formula:
(A) $((Fa \vee Fb) \vee (Fa \& Fb))$
(B) $(Fa \& Fb)$
(C) $(Fa \& \neg Fb)$
(D) None of the above.

C. Proofs

8. What is wrong with the following proof?

1. $\forall x \exists y Fxy$	
2. Show $\exists y \forall x Fxy$	
3. $\neg \exists y \forall x Fxy$	AIP
4. $\forall y \neg \forall x Fxy$	QN, 3
5. $\forall x Fxa$	$\exists$E, 1
6. $\neg \forall x Fxa$	$\forall$E, 4

(a) Illegal bracketing and cancelling -- no contradiction below line 2.
(b) Misuse of QN on line 4.
(c) Misuse of $\exists$E on line 5.
(d) Misuse of $\forall$E on line 6.
(e) Nothing is wrong.

9. What would be a usable strategy for the next step in the following derivation:

1. $\forall x \exists y Rxy$
2. $\forall x \forall y(Rxy \rightarrow Ryx)$
3. Show $\forall x \exists y Ryx$
4. Show $\exists y Rya$

(a) Apply ∃E to line 4, and then apply ∀E with lines 1 and 2.
(b) Apply ∀E to line 1 (using 'a'), then apply ∃E to the result (using a new constant).
(c) Start a universal proof of $\forall x \forall y(Ryx \rightarrow Rxy)$.
(d) Apply ∃E ro line 1, getting $\forall x Rxb$.
(e) Apply ∀E twice to line 2, getting $Raa \rightarrow Raa$.

10. Which of the following **cannot** be derived (**in one or more steps**) from $a = f(b)$ and $b=a$?
(a) $a = f(a)$
(b) $b = f(b)$
(c) $f(a) = f(b)$
(d) $f(f(a)) = b$
(e) All of the above can be derived.

11. What's wrong with the following proof?

1. Show $\forall x \forall y \forall z(x\neq y \& y \neq z \rightarrow x \neq z)$	
2. Show $a\neq b \& b\neq c \rightarrow a\neq c$	
3. $a\neq b \& b\neq c$	ACP
4. Show $a\neq c$	
5. $a=c$	AIP
6. $a\neq b$	&E,3
7. $b=c$	=E,5,6
8. $b\neq c$	&E,3

(a) Illegal use of ACP.
(b) Illegal use of AIP.
(c) Illegal use of &E.
(d) Illegal useof =E.
(e) Nothing is wrong.

12. Consider $\forall x \forall y(x = y) \rightarrow (\exists z Fz \rightarrow \forall z Fz)$
(A) The formula is true in every structure and so a valid formula of QL.
(B) The formula is true in every structure but not a valid formula of QL.
(C) The formula is not true in every structure but it is satisfiable.
(D) The formula is false in every structure and so a contradictory formula of QL

13 Which of the following is not a valid formula of QL.
(A) $\forall x Fx \rightarrow \exists x Fx$
(B) $\forall x(Fx \rightarrow Ax) \rightarrow \exists x(Fx \& Ax)$
(C) $\forall x(Fx \vee \neg\neg Fx) \rightarrow \exists y Fy$
(D) $\forall x(Fx \vee \neg Fx)$

For questions 14 - 15:
For every formula A of QL there is a quantifier free formula A* equivalent in every structure S that verifies the following formula:

$\forall z(z=a \vee (z=b \vee z=c))$.

14. $\forall x\, Fx$ is equivalent in every such S to:
(A) Fa
(B) $Fa \& (Fb \vee Fc)$
(C) $Fb \& (Fa \& Fc)$
(D) $Fb \vee (Fa \vee Fc)$

15 $\forall x \exists y Rxy$ is equivalent in every such S to:
(A) $(Rab \vee Rac \vee Raa) \& (Rba \vee Rbc \vee Rbb) \& (Rca \vee Rcb \vee Rcc)$
(B) $(Rab \& Rac \& Raa) \vee (Rba \& Rbc \& Rbb) \vee (Rca \& Rcb \& Rcc)$
(C) $(Rab \& Rac \& Raa) \vee (Rba \& Rbc \& Rbb) \& (Rca \vee Rcb \vee Rcc)$
(D) $\neg\forall x \exists y \neg Rxy$