Phl 313K
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THIRD MAJOR EXAM

Proofs

I. Suppose a and b are sets. Prove that $a \subseteq b$ iff $a \cup b \subseteq b$.

II. $\bigcup\{\{a, b\}\} = \{a, b\}$

III. Let a and b be any sets. Show that $\wp(a) \cap \wp(b) \neq \emptyset$.

II Multiple Choice:

A. Set Theory Concepts

1. Which of the following is false?
(A) Every set is a subset of itself.
(B) The membership relation ($\in$) is transitive: $\forall x \forall y \forall z((x \in y \;\&\; y \in z) \rightarrow x \in z)$.
(C) The subset relation is transitive.
(D) The null set is a subset of every set.
(E) No Ur-element has any members.

2. What's wrong with the following would-be derivation?

1.	$\forall x \forall y(\{x,y\}=\{z: z=x \vee z=y\})$	A
2.	Show $\forall x\ x=\{a,b\}$	
3.	Show $\{z: z=a \vee z=b\}=\{a,b\}$	
4.	$\{a,b\}=\{z: z=a \vee z=b\}$	$\forall E^2$, 1
5.	$\{a,b\}=\{a,b\}$	=I
6.	$\{z: z=a \vee z=b\}=\{a,b\}$	=E, 4,5

(A) Incorrect use of assumption on line 1.
(B) Incorrect use of ∀E on line 4.
(C) You can't use =E on a line already containing the = sign.
(D) Incorrect application of =I on line 5.
(E) Incorrect universal proof on lines 2 and 3.

3. Let

$a=\{1,2\}$
$b=\{3,4,5\}$
$c=\{2,4,6,8\}$.

Which of the following is the set $(a \cup b) \cap c$?

(A) $\{2,4,6,8\}$
(B) $\{1,2,3,4,5,6,8\}$
(C) $\varnothing$
(D) $\{2,4\}$
(E) $\{1,2,4\}$

4. If we know that R is reflexive, symmetric and transitive, we know all of the following **except**:
(A) R is not irreflexive.
(B) R is antisymmetric.
(C) R is an equivalence relation.
(D) R is not intransitive.

5. Consider the relation $R=\{\langle x,y \rangle: x, y \in N \text{ and } y=x+2\}$. Which of the following properties does R have?
(A) Reflexive.
(B) Symmetric.
(C) Transitive.
(D) Intransitive.
(E) Connected.

6. Consider the relation $R = \{\langle x,y\rangle : x, y \in N \text{ and } y > x + 1\}$.
Which kind of ordering on N is R?
(A) Partial ordering (reflexive).
(B) Strict partial ordering.
(C) Linear ordering (reflexive).
(D) Strict linear ordering.
(E) None of the above.

7. Which of the following is true of the axiom of extensionality?
(a) It is used to prove that two Ur-elements are identical.
(b) It is used to prove that two Ur-elements are distinct.
(c) It is used to prove that two sets are identical.
(d) It is used to prove that two sets are distinct.
(e) It does none of the above.

8. What follows from the assumption that $a \in b$?
(a) b is a set.
(b) $b \neq \varnothing$.
(c) $\{a\} \subseteq b$.
(d) $\{a\} \in \wp(b)$
(e) All of the above.

9. What (if anything) is wrong with the following would-be proof?

(1) "I will show that for every set A and relation R, R is connected on A.
(2) Assume that $a \in A$, $b \in A$, and $\langle a,b\rangle \in R$.
(3) Then $\langle a,b\rangle \in R$ or $\langle b,a\rangle \in R$.
(4) Since a and b were arbitrary, it follows that $\forall x \forall y\, (\langle x,y\rangle \in R \vee \langle y,x\rangle \in R)$.
(5) Applying the definition of connectedness, it follows that R is connected.
(6) Since R was arbitrary, every relation is connected."

(A) Improper assumption at step 2.
(B) Invalid inference at step 3.
(C) Illegal completion of universal proof at step 4.
(D) Improper use of definition at step 5.
(E) Nothing is wrong -- the proof is correct.

10. Suppose $R \subseteq A \times A$. Which statement is true?
(A) If R is symmetric and transitive on $\Im(R)$, then R is an equivalence relation on $\Im(R)$.
(B) If R is symmetric and transitive on A, then R is an equivalence relation on A.
(C) If R is reflexive on $\Im(R)$, then R is reflexive on A.
(D) If R is well-founded on A, then it's reflexive on A.
(E) If R is connected on $\Im(R)$, then R is connected on A.