Summer 03
Talbot

INTRODUCTION TO SYMBOLIC LOGIC

Prof. Rob Koons
rkoons@mail.utexas.edu

Text:

Robert Koons and Daniel Bonevac, *Tools for Thinking* (Oxford, 2003)

Classware:

Socrates 2.0 and *Aristotle 1.0* for Windows.

Socrates 0.8.8 is available for the Macintosh platform.

Available Aristotle & Socrates Web pages:

www.utexas.edu/courses/plato/aristotle
www.utexas.edu/courses/socrates

Requirements:

Homework: 20%
Logic workshops: 10%
Exams: 50%
Term project: 20%

Why are Logicians so Funny?

Kierkegaard: the comic is the perception of a painless contradiction.

The logic of Groucho Marx

Logic is the study of the universal laws of truth and consequence.

Valid for everything from physics to poetry. Even law.

Traditionally, one of the seven liberal arts. The capstone of the seven.

Distinctive Emphases of the Course

1. Practical, real-world application.

2. Semantics (meanings) of the logical symbols -- not merely rules for manipulation.

3. Informal presentation of mathematical proofs.

4. Breadth: wide range of forms of reasoning.

5. Logic as a multi-purpose tool.

6. Use of computer for guided self-instruction.

COURSE OBJECTIVES

1. To become adept at using logic in the following ways:

(1) verifying the correctness of logical inferences.

(2) exposing hidden assumptions of inferences.

(3) raising pertinent secondary questions, questions that must be answered in the course of answering the principal question.

(4) exposing inconsistency in information and assumptions.

(5) recovering from inconsistency by locating questionable information.

(6) understanding and creating informal proofs in mathematics.

(6) discovering and explaining anomalies.

(7) using logic as an adjunct to argumentation.

2. To acquire the following skills of argumentation:

(1) identifying arguments.

(2) understanding and analyzing arguments.

(3) evaluating arguments for correctness and completeness.

(4) repairing incomplete or otherwise defective arguments.

(5) constructing new arguments.

3. To gain fluency in the language of symbolic logic through:

(1) translating English statements in to the language of symbolic logic.

(2) translating statements in symbolic logic into English.

(3) using semantic tables to analyze the logical implications of statements in the language of symbolic logic.

4. To gain basic competence in the following forms of reasoning:

(1) propositional logic.

(2) predicate logic with identity.

(3) modal logic (the logic of necessity and possibility).

(4) defeasible logic.

PART I: BASIC PROPOSITIONAL LOGIC

ARGUMENTS

Our first task in logic is that of *argument analysis*. Analysis is applied to a block of text or speech that is argumentative in nature.

Argumentative text -- informal presentation of one or more lines of reasoning.

Argument analysis -- spelling out explicitly the questions and inferences implicit in an argumentative text.

The skill of argument analysis is fundamental: it is needed for the evaluation, criticism, repair and construction of arguments.

Arguments consist of premises, conclusions, and logical inferences.

Premises -- a collection of statements, known or believed or assumed to be true The starting point of a process of reasoning.

Conclusion -- a statement whose truth is to be derived from the premises by a process of reasoning. The goal of the inquiry.

Statement -- the use of a sentence to attempt to state a fact or represent information about the world. A statement is either true or false, and never both.

Inference -- the process of making explicit what is implicit in a body of statements.

Deductive inference -- a truth-preserving inference: if the input is true, the output cannot possibly be false.

Some other important terms:

Validity: the property of a correct deductive inference; truth-preserving.

Implication: a set of sentences S implies (or entails) a sentence A just in case the argument S ∴ A is deductively valid.

Consistency: a set of statements S is consistent if it is (logically) possible for all of them to be true.

Inconsistency: a set of statements S is inconsistent if it is logically impossible for all of them to be true.

There is a very tight connection between validity and inconsistency:

> An argument $A_1.A_2, ...A_n$ ∴ B is deductively valid if and only if
>
> the set $\{A_1,A_2,...,A_n,\text{not-B}\}$ is inconsistent.

1.2 Some examples of logical reasoning:

G. K. Chesterton's "The Secret Garden", from *The Innocence of Father Brown*

Cast of characters:

Valentin -- chief of police in Paris

Lord & Lady Galloway -- English ambassador & wife

Lady Margaret Graham -- daughter of these

Dr. Simon -- French scientist

Father Brown

Commandant O'Brien -- of French Foreign Legion

Julius K. Brayne -- American millionaire

The mystery takes place in Valentin's home: an absolutely secure house with surrounding gardens. After dinner the guests disperse throughout gardens. Soon a body is discovered: the head is completely severed, neck and shoulders slashed. The face is recognized by Valentin -- it is that of an American thief, whose twin brother had just been guillotined. Julius Brayne has disappeared.

Dr. Simon asks:

(1) How did the victim enter the garden?

(2) How did Brayne leave the garden?

(3) Why were neck and shoulders of victim mutilated?

Argument Analysis

1. The body of the victim could not (in its entirety) have entered or been brought into the garden.

2. The only person who was in the garden and who is unaccounted for is Brayne.

æ3. The body is (at least in part) that of Brayne. [LI, from 1, 2]

4. The head is not that of Brayne, nor of anyone else who was in the garden.

æ5. The head and the body do not belong to the same person. [LI, from 3, 4] æ6. The head was brought into the garden. [LI, from 4]

7. The only person who could have brought a head into the garden is Valentin.

æ 8. Valentin brought the head into the garden. [LI, from 6, 7]

9. Whoever brought the head into the garden is the murderer.

æ 10. Valentin is the murderer. [LI from 8,9]

A second example: Socrates and the slave boy in the Meno.

1. The area of a square whose sides are 2 feet long is 4 sq. feet. [Premise]
2. The area of a square whose sides are 4 feet long is 8 sq. feet. [Premise]
3. A square whose sides are 4 feet long is equal in area to four squares whose sides are 2 feet long. [Premise]
4. The area of a square whose sides are 4 feet long is 16 sq. feet. [Logical Inference, from 1,3]
5. 8 = 16. [LI, from 2, 4]
6. $8 \neq 16$. [New premise]

Socrates then enables the slave boy to work out a range within which the true answer must lie:

1. If there are squares x, y, and z, with areas 4, 8 and 16 respectively, then the length of the sides of square y will be between the lengths of the sides of squares x and z. [IM]
2. The area of a square with sides 2 ft. long is 4 sq. feet. [IM]
3. The area of a square with sides 4 ft. long is 16 sq. feet. [IM]
4. The length of the sides of a square with area 8 sq. feet is between 2 and 4 feet. [LI, from 1, 2, 3]

The Table Method

To begin an analysis, we construct a table.

Here are a few of the basic facts about semantic tables:

- Statements on the table are assumed to be true.
- We place all the premises, plus the denial (or "negation") of the conclusion on the table.
- If these assumptions are compatible, then the argument is logically incorrect (at least, incomplete).
- If these assumptions are incompatible, then the argument is logically correct.

The process of argument analysis consists in making the information implicit in the premises and the conclusion explicit. This process involves logical inferences, which extend or create paths (vertical columns).

- Inference moves preserve truth.
- A path is a vertically arranged sequence of statements on the table.

Disjunctive information posits more than one alternative. For example, consider the statement.: either the butler did it or the maid did. There are two ways that this statement could be true. Inference moves applied to disjunctive information split the path into two parallel **sub-paths**.

Each table is a compilation of possibilities. Each possibility corresponds to one of the paths of the table.

- A **path** is **closed** if it contains logically incompatible bits of information. Closed paths are marked with an X.

In particular, if a single path in the table contains both the statement A and the statement 'It is not the case that A' (the negation of A), then that path does not represent a real possibility.

- **A table is closed** if every path in it is closed.

Consider the following argument analysis:

1. Either the butler did it or the maid did it.		(assumption)
2. The butler did not do it.		(assumption)
3. The maid did not do it.		(assumption)
4. The butler did it	The maid did it.	(From 1)
5. X		(contradiction)
6.	X	(contradiction)

The table is closed

We have already seen the rule of Disjunction Decomposition illustrated. Whenever a disjunction (a statement involving the connective 'or') occurs on a path in the left half of the table, we may split this path into two sub-paths, placing one-half of the statement (the part preceding the 'or') in the left sub-path, and the remaining half of the statement in the right sub-path.

The Modus Ponens rule concerns the unpacking of information implicit in an 'if..., then...' statement (called, in logic, a conditional statement). If on one path in the left side of the table we find both a statement of the form 'if A, then B' and another statement asserting simply 'A', then we may add the statement 'B' to the same path. For example, consider the following simple argument:

---

1. If I don't pay my taxes, I will go to jail.	(assumption)
2. I don't pay my taxes.	(assumption)
3. I will not go to jail.	(assumption)
4. I will go to jail.	(Modus Ponens, 1, 2)
5. X	(contradiction)

The table is closed.

In this argument there are two premises and a single conclusion. From premises 1 and 2, line 4 follows by Modus Ponens. [w1.03]

A third important inference rule is that of Conditional Decomposition. Consider the following argument:

---

1. If everyone is here, there is a quorum.			(assumption)
2. There is not a quorum.			(assumption)
3. Everyone is here.			(assumption)
4. Not everyone is here.		There is a quorum.	(CD, 1)
5.	X		(contradiction)
6.		X	(contradiction)

The table is closed.

LOGICAL ABBREVIATONS

¬: the negation, abbreviating the phrase 'not...'

→ : the conditional, abbreviating the phrase 'if..., then...'

&: the conjunction, abbreviating the phrase '...., and ....'

∨: the disjunction, abbreviating the phrase '..., or ....'

↔: the biconditional, abbreviating the phrase '...if and only if ...'

1. Any capital letter A, B, … Z, followed by any number of primes ('), is a well-formed sentence.

2. If $\mathcal{A}$ and $\mathcal{B}$ are well-formed sentences, so are: $\neg\mathcal{A}$, $(\mathcal{A}\&\mathcal{B})$, $(\mathcal{A}\vee\mathcal{B})$, $(\mathcal{A}\rightarrow\mathcal{B})$, and $(\mathcal{A}\leftrightarrow\mathcal{B})$.

All of the well-formed sentences of sentential logic can be formed by repeated applications of rules 1 and 2.

Thus, the following strings of characters are not well-formed:

$(\neg A)$	$\neg(A)$
(A)	$((A)\vee(B))$
$A\&B$	$(A\&B) \rightarrow A$

In contrast, the following are well-formed sentences of sentential logic:

$$((A\&B)\vee(B\rightarrow A))$$

$$\neg\neg(\neg A\leftrightarrow \neg (B\vee\neg A))$$

$$\neg\neg\neg\neg A$$

$$(A\vee(A\&(A\leftrightarrow A)))$$

Some principles for translation:

Not:

The Haitian President does not authorize invasion.

¬(The Haitian President authorizes invasion)

¬D

And:

The UN and the OAS have authorized an invasion.

(The UN has authorized an invasion) & (The OAS has authorized an invasion)

A & B

The UN has authorized both an embargo and an invasion.

(The UN has authorized an embargo) &
(The UN has authorized an invasion)
C & B

Logical equivalents to 'and':

but, although, however, moreover, yet

Non-restrictive relative clauses:

The UN, which has authorized an embargo, has also authorized an invasion.

Or:

Either the UN or the OAS is the appropriate authority.

(The UN is the appropriate authority) ∨ (The OAS is the appropriate authority)

$E \vee F$

Logical equivalents to 'or':

unless, or else

Inclusive vs. Exclusive

If-then:

If the invasion is authorized by the appropriate authority, then the US should invade.

(The invasion is authorized by the appropriate authority) → (The US should invade)

$G \rightarrow H$

'If' and 'only if'

The US should invade if the invasion is authorized by the appropriate authority.

(The invasion is authorized by the appropriate authority) → (The US should invade)

$G \rightarrow H$

The US should invade only if the invasion is authorized by the appropriate authority.

(The US should invade) → (The invasion is authorized by the appropriate authority)

$H \rightarrow G$

Antecedent: the first half of a conditional, before the →

Consequent: the second half of a conditional, after the →.

The antecedent is asserted to be a **sufficient** condition of the consequent.

The consequent is asserted to be a **necessary** condition of the consequent.

If you take 180 credits, then you graduate.

C → G

You graduate only if you take 180 credits.

G → C

If you smoke, then you have a high risk of cancer.

You smoke only if you have a high risk of cancer.

$S \rightarrow C$

'If and only if'

The US should invade if and only if the invasion is authorized by the appropriate authority.

(The US should invade if the invasion is authorized) & (The US should invade only if the invasion is authorized)

((The invasion is authorized) → (US should invade)) & ((US should invade) → (The invasion is authorized))

$(G \rightarrow H) \& (H \rightarrow G)$

$G \leftrightarrow H$

Complex Sentences with more than one operator

Need parentheses:

The US should invade if the invasion is authorized; the invasion is in our national interest.

The US should invade if the invasion is authorized and the invasion is in our national interest.

$(G \rightarrow H) \& K$

$(G \& K) \rightarrow H$

Similarly, $A \& B \vee C$ is ambiguous:

$(A \& B) \vee C$ or $A \& (B \vee C)$.

1. Any capital letter A, B, ... Z, followed by any number of primes ('), is a well-formed sentence.

2. If $\mathcal{A}$ and $\mathcal{B}$ are well¬formed sentences, so are: $\neg\mathcal{A}$, $(\mathcal{A}\&\mathcal{B})$, $(\mathcal{A}\vee\mathcal{B})$, $(\mathcal{A}\rightarrow\mathcal{B})$, and $(\mathcal{A}\leftrightarrow\mathcal{B})$.

All of the well-formed sentences of propositional logic can be formed by repeated applications of rules 1 and 2.

$(\neg A)$

$\neg(A)$

(A)

$((A)\vee(B))$

$A\&B$

$(A\&B) \rightarrow A$

$((A\&B)\vee(B\rightarrow A))$

$A\vee(A\&(A\leftrightarrow A)))$

$\neg\neg\neg\neg A$

$\neg\neg(\neg A\leftrightarrow \neg (B\vee\neg A))$

There should always be exactly as many left parentheses in a sentence as there are right parentheses, and there should be exactly one pair of parentheses for each binary connective.

Truth Functions

A	¬A	A	B	(A&B)
T	F	T	T	T
F	T	T	F	F
		F	T	F
		F	F	F

A	B	(A∨B)	A	B	(A→B)
T	T	T	T	T	T
T	F	T	T	F	F
F	T	T	F	T	T
F	F	F	F	F	T

Material conditional (→) is essentially disjunctive:

If A, then B ≈ Either not A or B

$(A \rightarrow B) \approx (\neg A \vee B)$

$(\neg A \vee B)$ contains the minimal amount of information consistent with the claim that A is in some sense sufficient for B.

If $(\neg A \vee B)$ is true, and A is true, then B must be true (since ¬A will be false).

For mathematical purposes, this is all the information we need.

A	B	$(A \leftrightarrow B)$	$(A \leftrightarrow \neg B)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	T	F

$\neg (A \leftrightarrow B)$ is equivalent to $(A \leftrightarrow \neg B)$

Equivalences for complex denials

A	B	$\neg(A \& B)$	$(\neg A \vee \neg B)$
T	T	F	F
T	F	T	T
F	T	T	T
F	F	T	T

A	B	$\neg(A \vee B)$	$(\neg A \,\&\, \neg B)$
T	T	F	F
T	F	F	F
F	T	F	F
F	F	T	T

A	B	$\neg(A \rightarrow B)$	$(A \,\&\, \neg B)$
T	T	F	F
T	F	T	T
F	T	F	F
F	F	F	F

A	$\neg A$	$\neg\neg A$
T	F	T
F	T	F

Evaluating argument forms:

1. List the sentence letters appearing in the argument form.
2. Beneath them, list all possible interpretations.
3. List each premise formula, and then the conclusion formula.
4. Compute the value of each formula.

P	Q	P	(P	→	Q)	∴ Q
T	T	**T**	T	**T**	T	**T**
T	F	T	T	F	F	F
F	T	F	F	T	F	T
F	F	F	F	T	F	F

P	Q	¬	(P	&	¬	Q)	∴ P	↔	Q	
T	T	**T**	T	F	F	T	T	**T**	T	
T	F	**F**	T	T	T	F	T	**F**	F	
F	T	**T**	F	F	F	T	F	**F**	T	*
F	F	**T**	F	F	T	F	F	**T**	F	

What is Logic? Logical Inference?

Psychologism: logic = laws of human thought.

Logic is normative, not merely descriptive.

Logic makes explicit what is implicit.

What does 'implicit information' mean?
What does 'information' mean?

Logically valid inference -- impossible for the premises to be true while the conclusion is false.

What does impossible mean here?

This is gold.	This is water
This is element 59.	This is H_20.

Marie has been decapitated.
Marie is dead.

Distinguish: logical possibility &
physical possibility.

Imaginable possibility?

There is a 1000-sided figure.

What about:

John is a bachelor.	Mary is an optham.
John is unmarried.	Mary is an MD.

Ludwig Wittgenstein, *Tractatus Logico-Philosophicus* .

Atomic propositions: either true or false.

Logically independent.

Each assignment of truth-values to all atomic sentences is a possible world, a point in logical space.

A	B
T	T
T	F
F	T
F	F

Truth-functional proposition: a proposition whose truth-value (in any world) is a function of the truth-values of certain atomic propositions in that world.

An argument is logically valid if and only if the conclusion is true in every world in which all the premises are true.

Straightforward mathematical question.

Positive and Negative Contexts

The following introduce negative contexts:

¬________	(______→)
Denial	Antecedent of →

A part of a sentence occurs negatively in that sentence if and only if it occurs within an odd number of nested negative contexts.

In the following sentences, the sole occurrence of A occurs negatively, and the sole occurrence of B occurs positively:

$\neg A$	$(A \rightarrow B)$	$((C \vee A) \rightarrow (B \& D))$
$\neg(B \rightarrow A)$	$\neg(\neg A \rightarrow B)$	$((B \rightarrow A) \rightarrow C)$

Strengthening and weakening sentences:

To weaken a sentence logically, do the following within **positive** contexts:

(1) Delete conjuncts.

(2) Add disjuncts.

(3) Conditionalize: e.g., replace A with (B→A).

To weaken a sentence logically, do the following within **negative** contexts:

(1) Add conjuncts.

(2) Delete disjuncts.

(3) Deconditionalize: e.g., replace (B→A) with A.

To **strengthen** a sentence logically, do the following within **positive** contexts:

(1) Add conjuncts.

(2) Delete disjuncts.

(3) Deconditionalize.

To strengthen a sentence logically, do the following within **negative** contexts:

(1) Delete conjuncts.

(2) Add disjuncts.

(3) Conditionalize: e.g., replace A with $(B \rightarrow A)$.

Strengthen the following sentences:

1. $((A \& B) \rightarrow C)$
2. $(B \rightarrow D)$
3. $((A \vee D) \rightarrow (C \vee B))$

Now weaken the same sentences:

Principles of Argument Analysis

I. Distinguish between what is and what should be.
Represent by means of different sentences.

II. Distinguish between negation and opposition.

III. Find a valid argument whenever possible.
Even if the argument is invalid —— you should find a valid argument in the vicinity of what is said.

IV. Eliminate superfluous information in the premises and unnecessary weakness in the conclusion.
Experiment with eliminating or weakening premises: if the argument remains valid, make the change.
Try to make the conclusion as strong as possible —— so long as argument remains valid.

V. Separate separate arguments.
If you have two sets of premises, each of which would separately close the table, separate them into two arguments.

I. Distinguishing what is and what should be:

"College football should not have a national play-off, because if it does, regular season play will be diminished in importance, and the regular season should not be diminished."

Attempted analysis:

P: College football should have a national play—off.

R: Regular season play will be diminished in importance.

Symbolic argument:

$(P \rightarrow R), \neg R \therefore \neg P$ (w3.01)

What's wrong: a confusion of "should" and "will".

Q: College football **will** have a national play-off.

Improved analysis: $(Q \rightarrow R)$, $((Q \rightarrow R) \rightarrow \neg P) \therefore \neg P$

Even better:

C: A national play-off would cause a diminution of the importance of regular season play.

Argument: $(C \rightarrow \neg P), C \therefore \neg P$

"Something should be done about population control, because, if nothing is done, a famine will occur, and famines should not occur."

Attempted analysis:

D: Something should be done about population control.

F: Famine occurs.

Argument: $(\neg D \rightarrow F), \neg F \therefore D$

Another confusion of “should” and “will”.

Better analysis:

N: Doing nothing about population control will result in famine.

Argument: N, $(N \rightarrow D)$ ∴ D

Also: distinguish between "has" and "needs":

"The penal system needs more money." vs.

"The penal system has more money."

II. Distinguish between negation and opposition.

L: you are a lover.

¬L means: "you are not a lover"

It does **not** mean:

You are a beloved.

You are a hater.

III. Find a valid argument wherever possible.

Abbreviation:

G: The federal government is running a budget deficit.

M: The money supply is increasing.

V: The total amount available for investments is stable.

P: Interest rates will increase.

The symbolic argument:

G

$(\neg M \to V)$

$((G \& V) \to P)$

$\therefore P$ (w3.02)

What premise needs to be added to make the table close?

IV. Eliminate superfluous premises & unnecessary weakness in conclusion.

A: US Soldiers & civilians might get hurt.
B: Plan is a disaster.
C: Clinton disarms Cedras.
D: Cedras still running Haiti.

$(A \rightarrow B)$
$(C \vee A)$
$(\neg C \,\&\, D)$
$\therefore B$ (w3.03)

Which premise can be weakened without depriving the argument of validity?

S: The nation remains under a laissez faire system.

P: The nation will prosper.

H: The community works to solve health care.

O: The community works as an efficient organization.

1. $(P \rightarrow (O \,\&\, H))$
2. $(\neg P \vee O)$
3. $(S \rightarrow (\neg O \,\&\, \neg H))$

$\therefore (S \rightarrow \neg P)$ (w3.04)

Which premises can be weakened or deleted altogether?

G: animals have a divine right to life.

A: animals have a right to life.

L: animals have a right to life granted them by humans.

K: human laws prevent us from killing animals for food or sport.

1. $(G \rightarrow A)$
2. $(L \ \& \ \neg G)$
3. $(A \rightarrow K)$
4. $\neg K$

$\therefore \neg A$ (w3.05)

What redundancies in the premises can we identify?

Can we re-interpret the premises in such a way to make facts about G and L relevant?

A better interpretation:

1. $(A \rightarrow (G \vee L))$
2. $(\neg G \ \& \ \neg K)$
3. $(L \rightarrow K)$

$\therefore \neg A$ (w3.06)

V. Separate separate arguments.

1. $(A \rightarrow B)$
2. $(C \rightarrow B)$
3. $(A \ \& \ C)$

$\therefore B$ (w3.07)

Break this into two separate arguments for B.

1. $(A \vee D)$
2. $(\neg D \rightarrow B)$
3. $\neg(A \vee B)$

$\therefore D$ (w3.08)

The "Paradoxes" of Material Implication

The following argument forms are valid:

$A \mathrel{.\!\because\!.} (B \rightarrow A)$

$\neg B \mathrel{.\!\because\!.} (B \rightarrow A)$

Remember: $(B \rightarrow A) \equiv (\neg B \vee A)$

Cases where "→" is a bad translation of "if...then":

A. Denied conditionals.

M: The media is biased in favor of the candidate.

E: The candidate wins.

English sentences:

"The fact that the media is biased in favor of the candidate is not by itself sufficient to guarantee that the candidate will win."

"It is not the case that if the media is biased in favor of a candidate, that candidate will win."

Mistranslation: $\neg(M \rightarrow E)$

This is equivalent to: $(M \& \neg E)$

Back translation:

"The media is biased in favor of the candidate, and the candidate won't win."

In such cases, it may be better to translate the conditional by means of a simple sentence letter, like C, and represent the denied conditional as ¬C.

In chapter 5, we will introduce some new alternatives.

B. The conclusion is a conditional, and the premises include the information that the antecedent is false.

P: There is a national play-off.

Q: Regular season play will be diminished.

R: Every game counts.

1. $\neg P$

2. $(P \rightarrow \neg R)$

$\therefore (P \rightarrow Q)$

Premise 2 is redundant. The conclusion follows from premise 1 alone.

Solution: replace premise 1 by $(\neg R \rightarrow Q)$

C. The conclusion is a conditional, and the premises include the information that the consequent is true.

G: Gore will be re-elected in 2000.

H: Congress passes major health care reform in 98.

M: Gore receives a majority vote in the Electoral College in 2000.

"Gore won't be re-elected in 2000, since Congress won't pass health care reform in 98. So, even if Gore receives a majority vote in the EC, he won't be re-elected."

1. $(\neg G \,\&\, \neg H)$

$\therefore (M \rightarrow \neg G)$

This crazy argument turns out to be logically valid, when translated in this way. The fundamental problem is this: the conclusion is much too weak, given these conclusions. We can strengthen the conclusion to simply ¬G, which eliminates the paradoxical suggestion that even the Electoral College couldn’t elect Gore.