Modal and Conditional Logic

Modal logic is the logic of necessity, possibility and contingency. Conditional logic is the logic of typicality or normality.

$\Box\mathcal{B}$: $\mathcal{B}$ is necessary, $\mathcal{B}$ is true in every possible world.

$\Diamond\mathcal{B}$: $\mathcal{B}$ is possible, $\mathcal{B}$ is true in some possible world.

$\mathcal{B}$: $\mathcal{B}$ is true in the actual world.

$\mathcal{A} \mathrel{\Box\!\!\rightarrow} \mathcal{B}$: normally, if $\mathcal{A}$ is true, then so is $\mathcal{B}$. $\mathcal{B}$ is true in all normal $\mathcal{A}$-worlds.

$\mathcal{A} \mathrel{\Diamond\!\!\rightarrow} \mathcal{B}$: it's not the case that normally, if $\mathcal{A}$ is true, then $\mathcal{B}$ is false. $\mathcal{B}$ is true in some normal $\mathcal{A}$-worlds.

From Interpretations to Models

In propositional logic, we defined an argument as valid if every interpretation that made the premises true also made the conclusion true. I.e., an argument is valid if we can find no interpretation in which the premises are true and the conclusion false.

Each interpretation consisted of an assignment of truth values (T or F) to each simple sentence.
An interpretation can be thought of as a simple representation of a *possible world*.

In moving from simple propositional logic to modal logic, we want to enrich our set of logical symbols, enabling us to make reference to alternative possibilities. This is what ◇ and □ do.

In place of interpretations, we must consider representations of reality that are capable of representing the reality of alternative, parallel possibilities.

These representations are called "possible-worlds models".

Where propositional interpretations were one-dimensional (corresponding to a single row of a truth table), possible-worlds models are two dimensional.

Here is a simple example. Consider the argument: $\Diamond A \;\therefore\; A$

E.g., I am possibly rich. Therefore, I am rich.

This is clearly invalid. Here is a model that demonstrates its invalidity:

	A
w0	F
w1	T

This model consists of two worlds, w0 and w1. **To each world, we assign an interpretation of all of the sentence letters in the argument.**

In world 0, the sentence A is false, and in world 1 the sentence A is true.

Consider what happens in world 0 in this model.

The sentence ◇A is true in world 0, because there is another world in the model (world 1) in which A is true. Simultaneously, A itself is false at world 0. Hence, the premise is true at w0 in this model, and the conclusion is false at this same world in this model. So, the argument is invalid.

We say that an argument is valid in model logic if (and only if) every world in every model which makes the premises true makes the conclusion true as well.

Inspecting all possible models can be laborious. If there are n sentence letters in an argument there are $2^{2^n} - 1$ different models. For an argument with 2 letters, there are 15 models, with 3 letters 255 models, and with 4 letters about 64,000 models.

Translation.

To excel at anything, you *must* practice.
Practice is a *necessary condition* of excelling.
$\Box(Eab \rightarrow Pab)$

John *can* finish the race.
$\Diamond Fjr$

It's *possible* that there is life on Mars.
$\Diamond Lm$

The Prof's lecture was incomprehens*ible* to me.
$\neg\Diamond Cal$

If the strip has turned pink, acid *must* be present.
$(P \rightarrow \Box A)$

Whales are mammals. [Strict necessity]
$\Box(W \rightarrow M)$

Whales are large. [Normality]

$(W \Box\!\!\rightarrow L)$

Whales can be dangerous.

$(W \Diamond\!\!\rightarrow D)$

$\Diamond(W \& D)$

Most classes are large.

$(C \Box\!\!\rightarrow L)$

Few classes are large.

$(C \Box\!\!\rightarrow \neg L)$

Many classes are large.

$(Cx \Diamond\!\!\rightarrow L)$

Subjunctive conditionals

1. If Oswald had not assassinated Kennedy, someone else would have.
2. If Oswald did not assassinate Kennedy, someone else did.

1. $(\neg Aok \Box\!\!\rightarrow Aek)$
2. $(\neg Aok \rightarrow Aek)$

If the tank had been empty, my car wouldn't have started.
$(Ea \;\Box\!\!\rightarrow \neg Sc)$
If you were to study hard, you would do well.
$(Sa \;\Box\!\!\rightarrow Wa)$
If you were to study hard, you might do well.
$(Sa \;\Diamond\!\!\rightarrow Wa)$

Strengthening the antecedent of $\Box\!\!\rightarrow$
If you were to eat a breakfast, you would feel well.
If you were to eat a breakfast and get a headache, you would feel well.
If you were to eat breakfast, get a headache, and take 2 aspirin, you would feel well.
If you were to eat breakfast, get a headache, take 2 aspirin and take arsenic, you would feel well.

Applications

1. Action planning. Distinguishing between fixed and variable facts.

Rule of thumb: if I can deduce from what I know that p will happen, I don't need to worry about making p true.

Dudley Doright:

I am a hero. Nell is in distress.
Heroes always save damsels in distress.
Therefore, Nell will be saved.

Fallacious argument:

Hd, Dn, ((Hd&Dn) →Rdn), (Rdn → Sn)
∴ Sn

Distinguish between: Sn and □Sn. Dudley can deduce the first, not the second.

If I can deduce that □P, I needn't worry about making P true.

Invalid argument:

Hd, □Dn, □(Hd&Dn)→Rdn), □(Rdn→Sn)
∴ □Sn

Premises:			Conclusions:		
1.	H	(premise)	3.✓	(¬R→Sn)	(conclus
2.	(H→Sn)	(premise)	4.	¬¬R	(3 D→)
6.	Sn	(2,1 MP)	5.	Sn	(3 D→)

The table is closed.

H: I am a hero.
Sn: Nell will be safe.
R: I rescue Nell.

Premises:			Conclusions:		
1.	H	(premise)	3.✓	□(¬R→Sn)	(conclu
2.	(H→Sn)	(premise)	5.✓	(¬R→Sn) #∈1	(3 D□
4.	Sn	(2,1 MP)	6.	¬¬R ∈1	(5 D→
			7.	Sn ∈1	(5 D→

Premises:				Conclusions:		
1.	□(Un→Fn)		(premise)	3.✓	□(Un→Sn)	(conclusi
2.	□(Fn→Sn)		(premise)	4.✓	(Un→Sn) #∈1	(3 D□)
7.✓	(Un→Fn) ∈1		(1 D□)	5.	¬Un ∈1	(4 D→)
8.	(Fn→Sn) ∈1		(2 D□)	6.	Sn ∈1	(4 D→)
9.	¬Un ∈1	Fn ∈1	(7 D→)			
10.		Sn ∈1	(8,9 MP)			

The table is closed.

Reasoning about Knowledge:

We can let $\Box$ represent someone's state of knowledge – e.g., the jury's state of knowledge.

M: maid is guilty

B: butler is guilty

T: I testify truly

C: the jury convicts the butler

Argument:

$\Box(B \vee M), (T \rightarrow \Box\neg M), (\Box B \rightarrow C) \therefore (T \rightarrow C)$

Metaphysical Arguments

Anselm's Ontological Argument

G: God exists

P: a perfect being (omnipotent, etc.) exists

$\Box(G \leftrightarrow \Box P), \Diamond G \therefore (G \& P)$

Findlay’s Atheological Argument

$\Box(G \leftrightarrow \Box P), \Diamond\neg G \therefore (\neg G \& \neg P)$

Kripke’s Refutation of Mind-Brain Identity

$P\alpha$: α is a state of pain
$C\alpha$: α is a C-fiber firing (some particular neurological state)

$(Pa \rightarrow \Box Pa), (Cb \rightarrow \Diamond\neg Pb), (Pa \& Cb) \therefore \neg a{=}b$

Argument for the necessity of identity

$\Diamond\neg a{=}b \therefore \Box\neg a{=}b$

Deontic Logic: Reasoning about Morality and Legality

Let L represent the satisfaction of the demands of the law.

To say that A is **illegal**, we need:

$$\Diamond A \ \& \ \Box(A \rightarrow \neg L)$$

To say that A is **legally permissible**:

$$\Diamond(A \ \& \ L)$$

To say that A is **legally required**:

$$\Diamond\neg A \ \& \ \Box(L \rightarrow A)$$

The Punishment Paradox

M: Jones commits murder.
P: We punish Jones.
L: The law is followed.

Valid, but paradoxical:

$\Box P, \ \Box((L\&P) \rightarrow M) \ \ \therefore \ \ \Box(L \rightarrow M)$

Invalid:

$\Box P, \Box((L\&P)\rightarrow M) \ \therefore \ \Diamond\neg M \ \& \ \Box(L\rightarrow M)$

So, murder is not legally required.

Constructing Socrates tables for Modal, Conditional and Defeasible Logics

In the Socrates application, possible worlds are designated by means of numbers, from 0 to 99. The number 0 is always reserved for the actual (real) world. A □ on the table has universal force: it tells us something about all possible worlds. ◇ has existential.

When instantiating a formula having universal force, you may choose any world number whatsoever, including 0 or any number already occurring on the table. When instantiating a formula having existential force, you must choose a new number greater than 0.

On Socrates tables, some formulas are annotated by the Greek letter epsilon (∈), followed by a world number. When a formula occurs without such annotation (as, for instance, will the premises and conclusison), it is understood that the formula is being represented as true (or, on the right side, as false) in the actual world (world 0).

In order to close a path by contradiction, you must have contradictory formulas on the path with the same world-number labels.

Socrates Table Moves

1. Decompose Necessity
2. Decompose Possibility
3. Alter Denied Modality. Replaces − by $\Diamond\neg\mathcal{A}$, and replaces $\neg\Diamond\mathcal{A}$ by $\Box\neg\mathcal{A}$.

Modal Examples

1. $\Box A \mathrel{.\,\because\,.} \Box\Box A$
2. $\Diamond A \mathrel{.\,\because\,.} \Box\Diamond A$
3. $\Box(A\rightarrow B) \mathrel{.\,\because\,.} (\Box A\rightarrow\Box B)$
4. $\Box(A\vee B) \mathrel{.\,\because\,.} (\Box A\vee\Box B)$
5. $\Box A \mathrel{.\,\because\,.} A$
6. $\Box(A\&B) \mathrel{.\,\because\,.} (\Box A \;\&\; \Box B)$
7. $(\Box A\&\Box B) \mathrel{.\,\because\,.} \Box(A\&B)$

The Various Systems of Modal Logic: S5, S4, T and K

5. (◇A →□◇A)	S5 only
4. (□A → □□A)	S4, S5
T. (□A → A)	T,S4, S5
D. (□A → ◇A)	D, T, S4, S5
K. □(A→B)→(□A→□B)	K, D, T, S4, S5

Examples: doxastic and deontic lo

Doxastic logic: □A means "I believe that A."

Appropriate logic: KD45

Corresponding restriction on □L and ◇R:

Never apply to world 0

The following set is consistent in KD45:

□(Clinton is re-elected),

¬(Clinton is re-elected).

Logic of historical necessity, dynamics.
Appropriate logic: KT4.
Corresponding restriction on □ Decomp:

Apply only when you can when there is a world-creation path from the first world to the second.

Decision Problems

I. Practical Syllogism:

G: the ultimate goal
S: some intermediate sub-goal
A: some action

Premises: $\Box(G \rightarrow S), \Box(S \rightarrow A)$
Conclusion: $\Box(G \rightarrow A)$
Presupposes: $\Diamond G$

II. Satisficing argument

G: the goal -- a satisfactory result.
S: some intermediate sub-goal
A: some action

Premises: $\Box(A \rightarrow S)$, $\Box(S \rightarrow G)$
Conclusion: $\Box(A \rightarrow G)$
Presupposes: $\Diamond A$

III. Combined argument

Premises: $\Box(A \leftrightarrow S)$, $\Box(S \leftrightarrow G)$

Conclusion: $\Box(A \leftrightarrow G)$

The □→ conditional carries universal information of a sort. The information it carries is more limited than that carried by the □: (A□→B) means that B is true, not in absolutely all worlds, but in all normal A-worlds. The ◇→ conditional carries existential information of a special kind: (A◇→B) means that there is a normal A-world in which B is true.

The □ and ◇ of modal logic can be defined within conditional logic. We can take □A as an abbreviation for (¬A□→A) and ◇A as an abbreviation for (A ◇→A).

For this reason, Socrates has a rule (□ Definition) that enables us to replace (¬A □→A) by □A and (A □→ ¬A) by □¬A. The rule ◇ Definition allows us to replace (A◇→A) by ◇A.

Table Rules for Conditional Logic

Decompose Box-Arrow
Decompose Diamond-Arrow
Alter Denied Nonmonotonic
Box Definition
Diamond Definition

Decomposing ◇→ (existential force). You must introduce a new world. This becomes a normal antecedent world. (illustrate with 5.01)

Decomposing □→. 3 cases:
(1) You decompose to a new world, or to a world in which the antecedent is normal.
(2) You decompose to an old world in which some other formula is normal.
(3) You decompose to world 0.

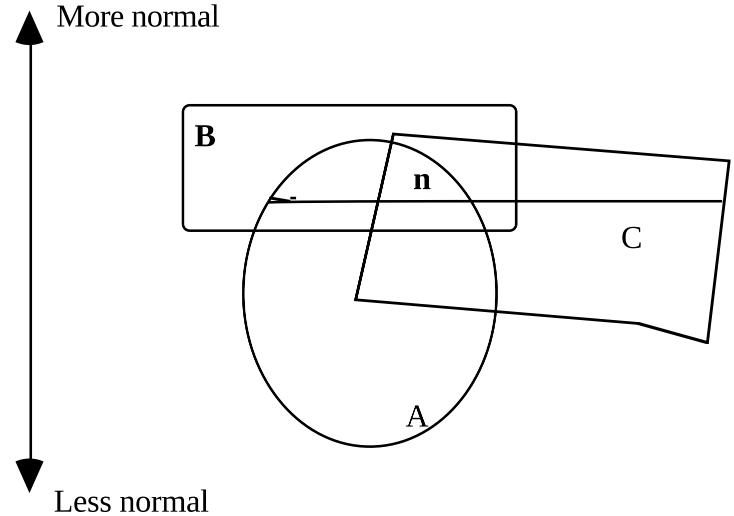
In case 1, the path splits in two.
Decompose (A □→ B) to world 1
(where 1 is new).
We get two paths:
(A & (A→B)) ∈ 1 on left, □¬A on right.

(illustrate 5.02)

In case 2, the path splits in two.
Decompose (A □→B) to world 1 (where some formula, say C, is normal at 1).
We get two paths:
(C & (A→B)) ∈1 in left; (C □→¬A) in right.
(w5.03)

Why does this happen?
If normal C worlds are not A-worlds, then nothing follows when applying (A□→B) to world 1. This possibility is the right path.

If some normal C-worlds are A-worlds, then if world 1 is an A-world, it must be a normal A-world. So, (A →B) is true at 1. (This goes into the left path)



Conditional Logic Examples

1. ¬◇P ∴ (P □→ Q)
2. (P □→ Q), ◇P ∴ ◇Q
3. (P □→ Q) ∴ (P □→ (P&Q))
4. (A ◇→ B), (B ◇→ A), (A □→ C) ∴ (B ◇→ C)

5. (A □→ B), (A □→ C) ∴ (A □→ (B&C))
6. (A □→ B) ∴ ((A&C) □→ B) [not valid]
7. (A □→ B), (B □→ C) ∴ (A □→ C) [not valid]

Some examples for further practice:
1. ((P∨Q) □→ R), ((P∨Q) ◊→ P) ∴ (P □→ R)
2. (P ◊→ Q), □(R → S), (P □→ R) ∴ ((P&Q) □→ S)
3. (P □→ Q), (Q □→ P), (P □→ R) ∴ (Q □→ R)
4. (P □→ Q) ∴ (P □→ (P □→ Q))

The following rules are very useful for the purposes of analyzing defeasible reasoning:

1. The Specificity Rule. (A □→ B), ((A&C) □→ -B) ∴ (A □→ -C)

2. Antecedent Substitution Rule. (A □→ B), (B □→ A), (A □→ C) ∴ (B □→ C)

3. Qualified Transitivity. (A □→ B), (B ◇→ A), (B □→C) ∴ (A □→ C)

4. Antecedent Shift. (A □→ B) ∴ (A □→ (A & B))

5. Weakening of the consequent. (A □→ ℬ) ∴ (A □→ ℬ'), where ℬ' is logically weaker than ℬ.

6. Consequent Conjunction. (A □→ B), (A □→ C) ∴ (A □→ (B & C))

Defeasible Reasoning

Deductive reasoning is foolproof: it is impossible for the premises to be true and the conclusion false.

Defeasible reasoning involves taking risk: jumping to a plausible conclusion.

With a **defeasibly correct** argument, it is possible, but extremely unlikely, for the conclusion to be false when the premises are true.

Deductive reasoning is **monotonic**:
If $\mathcal{A}_1,\ldots,\mathcal{A}_n \therefore \mathcal{B}$ is valid, so is $\mathcal{A}_1,\ldots,\mathcal{A}_n, \mathcal{A}_{n+1} \therefore \mathcal{B}$.

Defeasible reasoning is **non-monotonic**:
$\mathcal{A}_1,\ldots,\mathcal{A}_n \therefore \mathcal{B}$ may be d-correct, even though $\mathcal{A}_1,\ldots,\mathcal{A}_n, \mathcal{A}_{n+1} \therefore \mathcal{B}$ is not.

Deductive reasoning is a kind of monologue: making explicit what is already implicit in one's current beliefs.

Defeasible reasoning is dialectical: it can only be carried out effectively in the context of conversation, of the give and take of rebuttal and counter-rebuttal.

When using defeasible logic, we add only the premises to the table. We do not add the negation of the conclusion.

To test if the argument is defeasibly correct, we check if the conclusion is verified in every open path that remains.

A conclusion is verified in a path if it is logically entailed (using monotonic, deductive logic) by formulas occurring that path.

Examples:

If a path contains… Then … is verified.

A	(A∨B)
C, D	(C&D)
E	(F→E)
G, H	(G↔H)
¬J, ¬K	(J↔K)
(L→M), (M→L)	(L↔M)
¬¬N	N

Using the Close Path by Anomaly rule:

1. If, after completing analysis, there remains an open, anomaly-free path on the table, then all paths containing anomalies may be closed by the Close Path by Anomaly rule.

2. If, after completing analysis, a path on the left contains a **super-anomaly**, then that path may be closed by the Close Path by Anomaly rule.

Closing paths by anomaly is always a **comparative** judgment: one path is closed by anomaly because there are other paths either with no anomaly or with less weighty ones.

An **anomaly** is a pair of successive sentences in a path, marked by asterisks (*), generated by decomposing a □→ conditional to world 0.

Direct Explanation

Sentence $\mathcal{A}$ on path n **directly explains** a sentence $\mathcal{B}$ if and only if either:

(1) either ($\mathcal{A} \Box\!\!\rightarrow \mathcal{B}$) or ($\mathcal{A} \Diamond\!\!\rightarrow \mathcal{B}$) occur on path n (or one of these is entailed by the formulas in path n), or

(2) $\mathcal{A}$ deductively implies $\mathcal{B}$ (i.e., $\mathcal{B}$ is logically weaker than $\mathcal{A}$).

Definition of “Expanation”.

A sentence $\mathcal{A}$ on path n **explains** a sentence $\mathcal{B}$ if and only if either:

(1) $\mathcal{A}$ directly explains $\mathcal{B}$ on path n, or

(2) there is a chain of direct explanations on path n starting with $\mathcal{A}$ and ending with $\mathcal{B}$.

To illustrate, suppose the following formulas occur together on a path on a table:

(A□→B)
(B□→C)
(D◇→B)

On this path, we can say the following:

1. A explains: A, B, C, (C∨E).
   A explains itself, since it implies itself. A explains B because of the conditional (A□→B). A explains C, because A explains B and B explains C. C explains (C∨E) because C implies it, so A explains (C∨E) as well.

2. (A&E) explains: (A&E), A, B, C.
   (A&E) explains A because A is stricly weaker (implied). (A&E) explains B because A explains it, and (A&E) explains C because B explains it.
3. B explains: B, C, (B∨F), (C∨G).
   B explains (B∨F) because B implies it. B explains C because of the conditional (B □→ C). B explains (C∨G) because C explains it.
4. D explains: D, B, C, (C∨G).
   D explains B because of the conditional (D ◇→ B). It explains C because B explains it, and it explains (C∨G) because C explains it.
5. (D&G) explains: (D&G), D, B, C, (C∨F).
   (D&G) explains D because it implies it, and the other sentences because they are all explained by D.

Definition of Super-Anomalies.

Suppose $<\mathcal{A},\mathcal{D}>$ is an anomaly on path n. Then $<\mathcal{A},\mathcal{D}>$ is a **super-anomaly** on path n if there is a path m and, for **every** anomaly $<\mathcal{B},\mathfrak{I}>$ on path m, $<\mathcal{B},\mathfrak{I}>$ is explained by $\mathcal{A}$ (that is, $\mathcal{A}$ explains both $\mathcal{B}$ and $\mathfrak{I}$).

QPL (Qualitative Probability Logic)

Close a path by anomaly whenever:

1. the path contains an anomaly $<\mathcal{A},\mathcal{B}>$ (marked by an asterisk);
2. the table is complete with respect to prepositional & modal conditional logic;
3. either: there is at least one open path on the table that contains **no** anomalies; **or** $<\mathcal{A},\mathcal{B}>$ is a **super-anomaly** on the path.

Since the modal conditional □→ carries probabilistic information, it is possible to show that the defeasible logic defined above has the following property: if $\mathcal{A}_1,...\mathcal{A}_n$ ∴ $\mathcal{B}$ is defeasibly correct, then the conditional probability of $\mathcal{B}$ on $(\mathcal{A}_1\&...\&\mathcal{A}_n)$ is infinitely close to one, for any approximately uniform probability measure. This means that if you have a high degree of confidence in the conjunction of the premises and you have no other relevant information, it is reasonable to have a high degree of confidence in the conclusion.

1. (A □→ B), A ∴ B

Notice in this example that decomposing the box-arrow on line 1 to world 0 (the real world) splits the path in two: on the left path is added an anomaly (consisting of the antecedent and the denial of the consequent, marked with asterisks), and on the right place is placed the corresponding material conditional. When the decomposition of the statements is completed, we are left with two paths: the first contains an anomaly, but the second does not. Consequently, we can close path 1 on the left by anomaly. The conclusion is verified in the remaining path, so the argument is successful.

2. (A □→ B), ¬B ∴ ¬A

Here again, line 1 is decomposed to world 0, resulting in split paths. The second path in turn decomposes into paths containing ¬A and B. The path containing B closes by contradiction, thanks to line 2. In the end, there are two open paths. The first contains an anomaly, and the second is anomaly-free. Consequently, we can apply the Close Path by Anomaly rule to path 1. The conclusion ¬A is verified in the remaining path, so the argument is defeasibly successful..

3. ((A&B) □→ C), A, ¬C ∴ ¬B

After all the statements are decomposed on this table, we are left with two open paths. Only the first of these contains an anomaly. Consequently, we can close path 1. The conclusion, ¬B, is verified in the remaining path, so the argument is successful.

4. (A □→ B), (B □→ C), A ∴ C

In this case, we are left with three open paths, only the first of which contains an anomaly. Thus, once again, the Close Path by Anomaly rule applies to path 1.

5. (R □→ ¬P), (Q □→ P), Q, R ∴¬P
[The Nixon Diamond]

This argument reflects an example discussed often in the AI literature, the so-called 'Nixon diamond '. Let R represent 'Nixon is a Republican', Q represent 'Nixon is a Quaker', and P represent 'Nixon is a pacifists'. Premise 1 abbreviates the statement, 'for the most part, Republicans are not pacifists'. Premise 2 corresponds to: 'for the most part, Quakers are pacifists'. In this case, we can conclude neither that Nixon probably is nor probably is not a pacifist. After decomposing the premises, we are left with two open paths.

Each of these paths contains an anomaly: Neither of these anomalies is a super-anomaly on this table, since Nixon's being a Republican doesn't explain why he is a Quaker, and his being a Quaker doesn't explain why he is a Republican. Therefore, we cannot apply the Close Path by Anomaly rule, and neither path can be closed. The conclusion cannot be verified in all of the remaining open paths, so the argument is not defeasibly correct.

6. $(P \square\rightarrow \neg F), (B \square\rightarrow F), (P \square\rightarrow B), B, P \therefore \neg F$ [The Penguin Problem]

This argument embodies another example from AI research: the 'Penguin pattern'. Think of B as stating that Tweety is a bird, P that Tweety is a penguin, and F that Tweety flies.

We know that penguins do not fly (line 1), bird do fly (line 2) and penguins are birds (line 3). After decomposing the premises, we are left with two open paths, 2 and 3. Both of these open paths contain anomalies. However, the statements P and ¬F in path 2 constitute a super-anomaly, since P explains every anomaly in path 3, namely, it explains the anomaly <B, ¬F> on path 3. Premise 1 tells us that penguins do not fly, and premise 3 tells us that penguins are birds. Thus, P enables us to explain the <B,¬F> anomaly, making <P,¬¬F> in path 2 a super-anomaly. We can apply the Close Path by Anomaly rule to path 2, and the the conclusion, ¬F, is verified in the remaining open path. The argument is defeasibly correct.

7. ((P&B)□→¬F), (B□→F), B, P ∴ ¬F

[Another version of the Penguin]
In this version of the Penguin pattern, we know that Tweety is a penguin and a bird, that penguin-birds do not fly, and that birds do fly. Once again, we reach a point at which there are two open paths, 2 and 3. Each of these paths contains an anomaly. The anomaly in path 2 is a super-anomaly, since the statement (P&B) explains the one and only anomaly in path 3.
We know, from line 1, that (P&B) explains ¬F.
We do not have any explicit statement in path 2 to the effect that (P&B) explains B, but we can easily see that B is deductively implied by (P&B). We can use the Close Path by Anomaly rule to close path 2, and the conclusion is verified in the remaining path.

Rebuttals of Defeasible Reasoning

A rebuttal is a claim or set of claims, lodged as an objection to some argument.

Rebuttals:

If $\mathcal{A} \therefore C$ is a successful (correct) defeasible argument, then $\mathcal{B}$ is a rebuttal to this argument iff:

(1) $\mathcal{B}$ is a set of statements, and
(2) $\mathcal{A}, \mathcal{B} \therefore C$ is not successful (not defeasibly correct).

Two kinds of rebuttals:

1. Simple rebuttals. Results in a stalemate: neither $\mathcal{A},\mathcal{B} \therefore C$ nor $\mathcal{A},\mathcal{B} \therefore \neg C$ is correct.
2. Overriding rebuttal: $\mathcal{A},\mathcal{B} \therefore \neg C$ is successful.

Two kinds of simple rebuttals:
1. Counter-arguments
2. Raising a red flag
    2a. Undercutting rebuttal (a special case of 2)

Examples:

Jones is a Democrat.
Democrats are liberals.
∴ Jones is liberal
$D, (D \Box\!\!\rightarrow L) \;\therefore L$

Simple Rebuttals:
1. Counterargument.
Jones is from Lubbock. People from Lubbock are not liberal.
$U, (U \Box\!\!\rightarrow \neg L)$

2. Raising a red flag

Democrats do not support the balanced budget amendment, but Jones does.

$(D \;\Box\!\!\rightarrow \neg B)$, B

Original argument:

$(D \;\Box\!\!\rightarrow L)$, $D \;\therefore L$

2a. Undercutting.

A Democrat who opposes Clinton might not be liberal. Jones opposed Clinton.

$((D\&O) \;\Diamond\!\!\rightarrow \neg L)$, O

From $(D \;\Box\!\!\rightarrow L)$ and $((D\&O) \;\Diamond\!\!\rightarrow \neg L)$, it follows deductively that $(D \;\Box\!\!\rightarrow \neg O)$.

So, this turns into a case of Red flag rebuttal:

O, $(D \;\Box\!\!\rightarrow \neg O)$

In general, the following inference is deductively valid:
$(A \Box\!\!\rightarrow B), ((A \& C) \Diamond\!\!\rightarrow \neg B) \therefore (A \Box\!\!\rightarrow \neg C)$

Similarly, if we have an undercutting rebuttal of the form $(C \Diamond\!\!\rightarrow \neg B)$ and $(C \Box\!\!\rightarrow A)$, we can use the validity of the following inference:

$$(A \Box\!\!\rightarrow B), (C \Diamond\!\!\rightarrow \neg B), (C \Box\!\!\rightarrow A)$$
$$\therefore (A \Box\!\!\rightarrow \neg C)$$

We can use this fact to convert undercutting rebuttals into simpler, red-flag rebuttals:

Argument	Undercutting rebuttal	Red-flag substitute
A, (A□→B) ∴ B	C, ((A&C)◇→¬B)	C, (A□→¬C)
A, (A□→B) ∴ B	C, (C◇→¬B), (C□→A)	C, (A□→¬C)
A, (A□→B) ∴B	C, (C◇→¬B), □(C→A)	C, (A □→¬C

1. Simple rebuttal

Example: Nixon is a Quaker, Quakers are pacificists.

a. Raising a red flag. Nixon is a Republican, and Quakers are not Republicans.

b. Undercutting. Nixon is a Republican, and many Quaker Republicans are not pacificists.

c. Counter-argument. Nixon is a Republican, and Republicans are not pacificists.

2. Overriding rebuttal.

Nixon is a Republican, and Quaker Republicans are not pacificists.

A few examples:

Argument	Minimal Rebuttal
1. $A, (A \Box\!\!\rightarrow B) \therefore B$	$(C \Box\!\!\rightarrow D), C, \neg D$
2. $A, (A \Box\!\!\rightarrow B) \therefore B$	$(A \Box\!\!\rightarrow D), \neg D$
3. $\neg B, (A \Box\!\!\rightarrow B) \therefore \neg A$	$(C \Box\!\!\rightarrow D), C, \neg D$
4. $\neg B, (A \Box\!\!\rightarrow B) \therefore \neg A$	$(C \Box\!\!\rightarrow B), C$

Argument	Counter-argument
5. $\neg B, (A \Box\!\!\rightarrow B) \therefore \neg A$	$(C \Box\!\!\rightarrow A), C$
6. $A, (A \Box\!\!\rightarrow B) \therefore B$	$(C \Box\!\!\rightarrow \neg B), C$

Overriding Rebuttals

Argument	Overriding Rebuttal
1. $(A \Box\!\!\rightarrow B), A \therefore B$	$((A \& C) \Box\!\!\rightarrow \neg B), C$
2. $(A \Box\!\!\rightarrow B), A \therefore B$	$(C \Box\!\!\rightarrow A)$, $(C \Box\!\!\rightarrow \neg B), C$
3. $(A \Box\!\!\rightarrow B), \neg B \therefore \neg A$	$(\neg B \Box\!\!\rightarrow A)$ (ex 8.03)

Fallacy of Accident

Merely pointing out that an argument is defeasible is not an effective form of rebuttal.

Merely listing exceptions to a rule does not invalidate the rule, or rebut its application to the present case.

The "Uncle Joe rebuttal."

Example:

It is widely claimed that high budget deficits cause high interest rates. That this is untrue can be seen from the fact that in the 1980's in America, budget deficits were high and climbing while interest rates were low and declining.

Turning this into a real rebuttal:

Higher budget deficits in the current situation will not cause higher interest rates, since the deficits are being used to finance strongly pro-investment tax cuts. Such tax cuts have an even stronger effect on interest rates than do budget deficits, and they cause the rates to fall.

Original argument:

$B, (B \mathbin{\Box\!\!\rightarrow} H) \;\therefore\; H$

Overriding rebuttal:

$(B \,\&\, C), (B \mathbin{\Box\!\!\rightarrow} H), ((B \& C) \mathbin{\Box\!\!\rightarrow} \neg H)$
$\therefore \neg H$

Ethical Reasoning

Churchill & Coventry

C1. Spares innocent lives □→ Right
C2. (Spares innocent lives & Seriously impairs war effort) □→ Not Right
C3. Evacuation spares innocent lives & Evacuation seriously impairs war effort
Conclusion: Evacuation is not right.

C4. Spares own child □→ Right

Sophie's Choice

Antigone

Antigone -- by Sophocles

It is wrong not to bury your brother.

Polyneices is my brother.

∴ It is wrong not to bury Polyneices.

Rebuttals:

1. Polyneices was a traitor. Typically, brothers are not traitors.

2. Polyneices was a traitor. Traitors should not be buried.

3. Polyneices was a criminal. In many cases, it is not wrong not to bury your brother, if he's a criminal.

4. Polyneices was a traitor. It is not wrong not to bury a traitor, even if he's your brother.

Legal Reasoning: McLoughlin v. O'Brian

M1. Injury □→ Liable
M2. (Injury & Not physical) □→ Not liable
M3. (Injury & Not physical & Caused by carelessness) □→ Liable
M4. (Injury & Not physical & Caused by carelessness & Unforeseeable) □→ Not liable
M5. (Injury & Not physical & Caused by carelessness & Compensation would impose morally disproportionate financial burden) □→ Not liable

Uses of Logic in Law:

1. Analyzing the reasoning of others

2. Problem-solving:

    a. Given rules, finding facts that would support a conclusion.

    b. Given cases, finding a set of rules that would complete the arguments successfully.

Case-based Induction

A $\therefore$ L
B $\therefore$ ¬L
A, B $\therefore$ ¬L
A, C $\therefore$ L
B, C $\therefore$ L
A, B, C $\therefore$ L

LSAT example:
Some people fear that our first extraterrestrial visitors will not be the friendly aliens envisaged in popular science fiction movies, but rather hostile invaders bent on global dictatorship. This fear is groundless. Any alien civilization that makes it to our planet must have acquired the wisdom to control war, or it would have destroyed itself long before contacting us.
The author bases the argument on which of the following assumptions?
F: Aliens will be friendly to us.
W: Aliens have acquired the wisdom to control war.
C: Aliens contact us.
D: Aliens destroyed themselves before contacting us.

$(\neg W \;\Box\!\!\rightarrow \neg C) \;\therefore\; (C \rightarrow F)$

$(\neg W \;\Box\!\!\rightarrow \neg C),\quad \mathbf{(W \;\Box\!\!\rightarrow F)} \;\therefore\; (C \rightarrow F)$

Scientific Reasoning

Problem-solving and scientific inquiry.

Every unsolved scientific problem can be modelled as an open table. The ultimate goal of the scientific inquiry is an answer to what we will call the **principal question**.

Successful inquiry consists of finding manageable **operational questions** : questions that can be directly answered through observation and testing, and answers to which will help to determine the answer to the principal question.

Scientific problems are of two kinds: those that seek answers to Yes/No questions (factual inquiries), and those that seek answers to Why questions (explanatory inquiries).

In the case of factual inquiries, there are two possible answers to the principal question: the hypothesis is true, and the hypothesis is false.

We can construct a table by putting all the information we presently have that could be relevant to our inquiry on the table.

If all of the open paths verify the hypothesis, or if they all verify the negation of the hypothesis, then we already have an answer to the principal question. In most cases, when inquiry begins neither the hypothesis nor its negation will be verified.

We now have a familiar problem: what premises could be added to the table in order to make all the paths close except those that give a single, definite answer to our principal question. These formulas pose potential **operational questions**.

The operational question is simply: is this possible new premise true or false? If we can discover whether or not the possible new premise is true by direct observation or testing, then doing so may take us closer to a definitive answer to the principal question.

M: the suspect is the murderer

B: The suspect's blood is mingled with the victims' at the crime scene.

Incomplete argument: $(B \Box\!\!\rightarrow M) \therefore M$

Operational question: B?

Another possibility:

C: a police conspiracy planted the suspect's blood at the scene.

$B, (B \Box\!\!\rightarrow M), (B \Box\!\!\rightarrow \neg C) \therefore M$

New operational question: C?

G: the murderer's gloves fit the suspect.

$(\neg G \Box\!\!\rightarrow \neg M)\ \therefore \neg M$

Operational question: G?

S: the murderer's gloves have shrunk

$(\neg G \Box\!\!\rightarrow \neg M),\ ((\neg G \,\&\, S) \Diamond\!\!\rightarrow M)\ \therefore\ \neg M$

Operational questions: G? S?

Finding Scientific Explanations

Another important task of scientific inquiry is the construction of explanations. The phenomenon to be explained is called the *explanandum* . The facts that are adduced to explain this phenomenon are called the *explanans*. A scientific problem of this sort can also be modeled by means of an open table.

In the premises, one includes what those parts of the explanans that are already known: initial conditions, classifications of individuals involved, laws of nature (or legal/ethical rules).

The explanandum should be made the concusion of the argument.

The **principal question** of an explanation problem is: **why** is the conclusion is true?

By discovering premises that will cause the table to close, one uncovers **operational questions** , e.g.: is this premise true?

These operational question should satisfy two conditions:

1. It should be possible to discover the answer to the operational question by further observation or testing.

2. At least one answer would enable one to verify the conclusion in all open paths (thereby providing an explanation for the conclusion).

Here are two examples of explanation-seeking inquiries, both involving medical diagnosis.

1. Explaining fever.

Suppose that we observe a fever in the patient. Diagnosing the fever means looking for an explanation. An explanation would be a defeasible argument having the fact of fever as its conclusion and known or possible truths as its premises. The following is a simple example of an explanation, where 'F' stands for fever and 'B' for the existence of a bacterial infection.

$(B \;\Box\!\!\rightarrow F) \;\therefore\; F.$ Missing premise: B.

A more complicated case:

$(B \;\Box\!\!\rightarrow F), ((B \;\&\; C) \;\Box\!\!\rightarrow \neg F), ((B \& C \& D) \;\Box\!\!\rightarrow F), C \;\therefore\; F$

Operational question: (B&D)?

Theory Confirmation and Disconfirmation

In empirical science, theories and hypotheses are not inferred deductively from axioms (as are theorems in mathematics). Instead, such hypotheses are confirmed or disconfirmed by the available evidence. We can use defeasible logic to model these forms of inference.

A scientific hypothesis is confirmed when it enables us to generate a risky, surprising prediction that is ultimately verified by observation or testing. For example, British astronomer Edmund Halley used Newton's theory of gravitation to predict the return of a large comet at regular intervals.

General Form of Hypothesis Confirmation:

B: background facts and theories.

P: observed phenomenon -- return of comet at predicted intervals

H: hypothesis:Newton's law of gravity

$B, P, ((B \& \neg H) \Box\!\!\rightarrow \neg P) \;\therefore\; H$

Possible rebuttal: $((B \& H) \Box\!\!\rightarrow \neg P)$

So, hypothesis confirmation presupposes:

$\neg((B \& H) \Box\!\!\rightarrow \neg P)$, i.e., $((B \& H) \Diamond\!\!\rightarrow P)$.

The phenomenon should be unsurprising (not necessarily predicted), given the hypothesis.

Another LSAT example:

Most major retail electronic chains experienced a dramatic increase in the amount of merchandies lost to shoplifting during the early 1980s. By 1986, however, all large chains had installed new anti-theft devices in their stores. Since this time, there has been a sharp decline in the number of shoplifting incidents ...

Which, if true, would most strengthen the argument?

(A)The average size of merchandise was now small enough to fit into a person's pocket.
(B) The average cost of electronic merchandise decreased since 1986.
(C) Each item in the store was marked with an unremovable security number.
(D) Shoplifting increased at single outlet retail electronics operations since 1986.
(E) Shoplifting increased in the retail industry overall since 1986.

H: Antitheft devices causes lower shoplifting rates.

B: Antitheft devices added to large chains in 1986.

E: Shoplifting rates in large chains declined since 1986.

Need an E' such that ((B&¬H) □→ ¬(E&E'))

Which answer would be most surprising, given B & ¬H & E?

Theory Repair: Explaining Anomalies

Scientific theories are not rejected the moment an anomaly involving the theory is discovered. However, a theory that is subject to unexplained anomalies is of little or no use in making predictions and generating explanations. In order to make a theory useful again, it is critically important that the existing model of the world be modified in order to explain any observed anomalies.

For example, the standard model of the solar system (when only seven planets were known), incuding Newton's theory of gravitation, generated false predications about the orbit of Uranus. As long as this false prediction remained unexplained, the entire model, and all predictions and explanations about the solar system based on that model, were thrown into doubt.

Let S represent the existence of seven of the planets of the solar system,

N represent Newton's theory of gravitation,
O represent the predicted orbit of Uranus, and A some other prediction based on S&N (say, some prediction about the future course of comets).

The following table demonstrates the fact that an anomaly concerning O blocks the generation of a prediction concerning A.

In order to place our model of the solar system back on firm footing, we must find a fact that enables us to explain the apparently erratic orbit of Uranus. This inquiry led to the discovery of a new planet: Neptune. Let 'P' stand for the discovery of this new planet. The modified model, consisting now of (S&N&P) enables us to explain ¬O, and we can now use this repaired model in generating new predictions and explanations, as illustrated by the following table.

7d7

Another LSAT example:

A psychologist once performed the following experiment. Subjects were divided into two groups: excellent chess players and beginning chess players. Each group was exposed to a position arising from an actual game. Not surprisingly, when asked to reconstruct the position from memory an hour later, the expert chess players did much better than the beginners. On a board where the pieces were placed in a position at random, however, the expert players were no better able to reconstruct the position from memory than were the beginners.

Which of the following explains the result?

H: Greater familiarity with chess improves memory of piece positions.

E: Experts better at remembering actual positions.

R: Experts better at remembering random positions.

(H □→ (E&R)), H, E, ¬R

Anomaly: (H □→ R), H, ¬R

Need a new assumption such that ((H&A) □→ ¬R), or a new hypothesis such that (H' □→ ¬R), (H' □→ E).

From Darwin's *The Origin of Species*.

Argument:
If a transitional form **a** had existed, we would observe fossils of **a**.
According to D's T of E, transitional form **a** did exist.
Therefore, we should observe fossils of **a** [Prediction].
$(Pa \Box\!\!\rightarrow Oa), Pa \therefore Oa$

Rebuttal: Many transitional forms became extinct very quickly. We would not observe fossils of short-lived forms.
$Ta, ((Pa \& Ta) \Diamond\!\!\rightarrow Sa), (Sa \Box\!\!\rightarrow \neg Oa)$
It follows: $(Pa \Box\!\!\rightarrow \neg Sa)$.

If the transitional form had existed we would have found some trace of it. According to the T of E, the transitional form did exist. So, we would expect to find some trace.

$(Pa \;\Box\!\!\rightarrow Ra), \;\; Pa \;\; \therefore Ra$

Rebuttal: The only possible trace of ancient life forms is that of fossils. Fossils are laid down infrequently, so many ancient life forms have left no trace.

$((La \& Pa) \;\Diamond\!\!\rightarrow \neg Ra), \;\; La$

Undercutting

Mill's Methods of Inferring Causes

Case-Based reasoning: Nature gives us the cases, we try to supply the missing rules.

A,B,C	$\therefore$ E
A, B, ¬C	$\therefore$ E
A, ¬B, C	$\therefore$ ¬E
A, ¬B, ¬C	$\therefore$ ¬E
¬A,B,C	$\therefore$ E
¬A, B, ¬C	$\therefore$ E
¬A, ¬B, C	$\therefore$ E
¬A, ¬B, ¬C	$\therefore$ E

Interpreting arguments: the principle of charity.

Choosing between:

$(A \rightarrow B)$ and $(A \,\Box\!\!\rightarrow B)$.

The latter admits of exceptions.

When in doubt, interpret argument as defeasible.

Exceptions: (1) mathematical proofs.

(2) appeals to truisms or definitions.