

## Integral Practice Worksheet

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### Some area problems

- Let  $f(x) = x^2 - x$ ,  $x \in [-2, 2]$ 
  - Sketch the graph of this function. (Hint: Find the roots.)
  - Shade in the region between  $f(x)$  and the  $x$ -axis.
  - Write an integral (or integrals) representing the area of this region.
  - Solve (if time permits).
  
- Let  $f(x) = \cos x$ ,  $g(x) = \sin x$ ,  $x \in \left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$ .  
Find the area between  $g(x)$  and  $f(x)$ .
  
- Consider the curves  $4x = 4y - y^2$  and  $4x - y = 0$ .  
Sketch the region bounded by these curves and find its area.  
(Hint: It may be easier to integrate with respect to one variable than the other.)

### U-substitution

1. 
$$\int \frac{dx}{(3+4x)^2}$$

4. 
$$\int \frac{\cos \sqrt{x}}{\sqrt{x}} dx$$

2. 
$$\int_0^{\pi} x^2 \cos x^3 dx$$

5. 
$$\int x^5 \sqrt{x^2+1} dx$$

3. 
$$\int \frac{\sec^2 x}{\sqrt{1+\tan x}} dx$$

### Derivatives of Integrals

1. 
$$\frac{d}{dx} \left( \int_{x^2}^3 \frac{\sin t}{t} dt \right)$$

3. 
$$\frac{d}{dx} \left( \int_x^{2x} \frac{1}{1+t^2} dt \right)$$

2. 
$$\frac{d}{dx} \left( \int_1^{x^2} \frac{dt}{t} \right)$$

4. 
$$\frac{d}{dx} \left( \int_{\tan x}^4 \sin(t^2) dt \right)$$